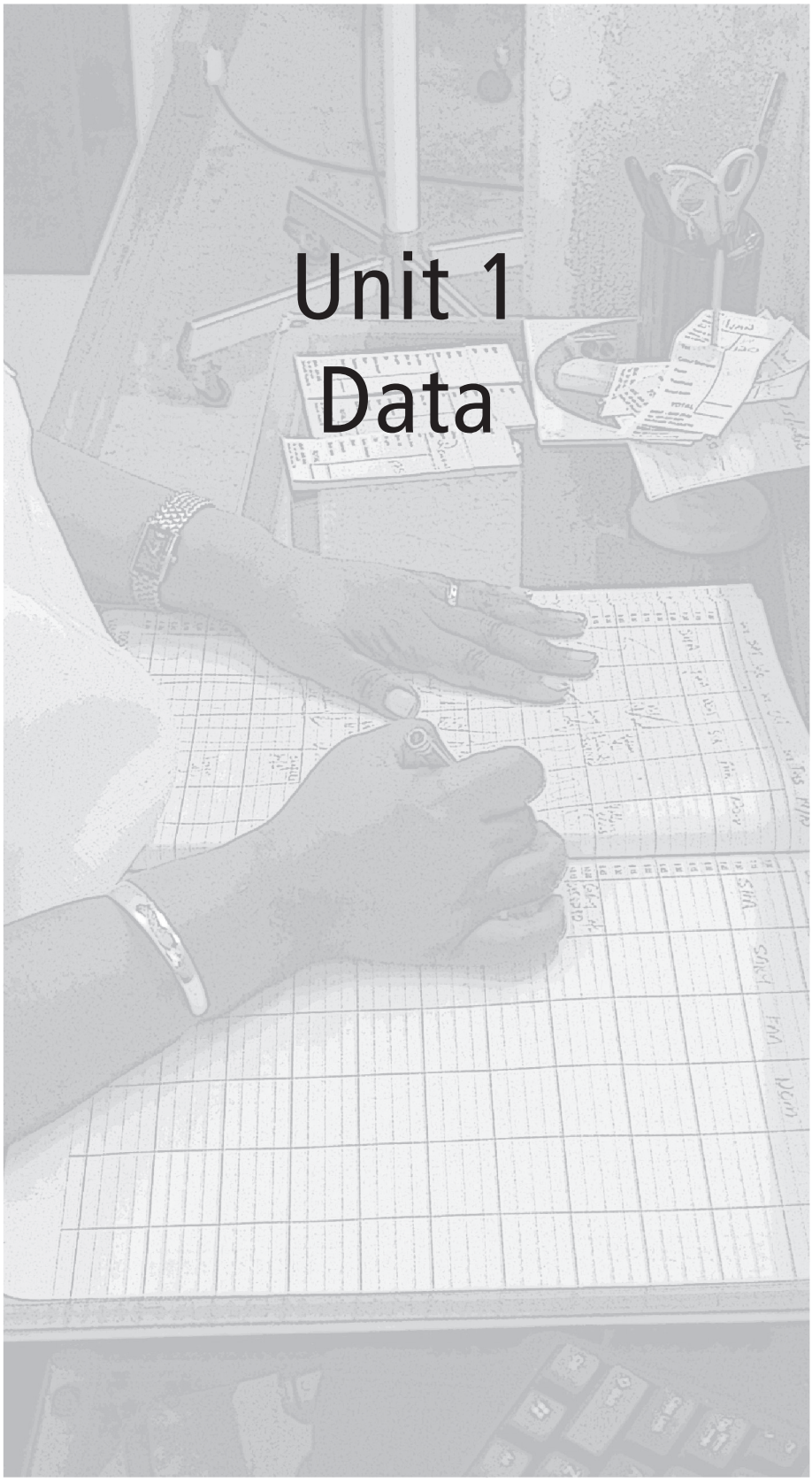
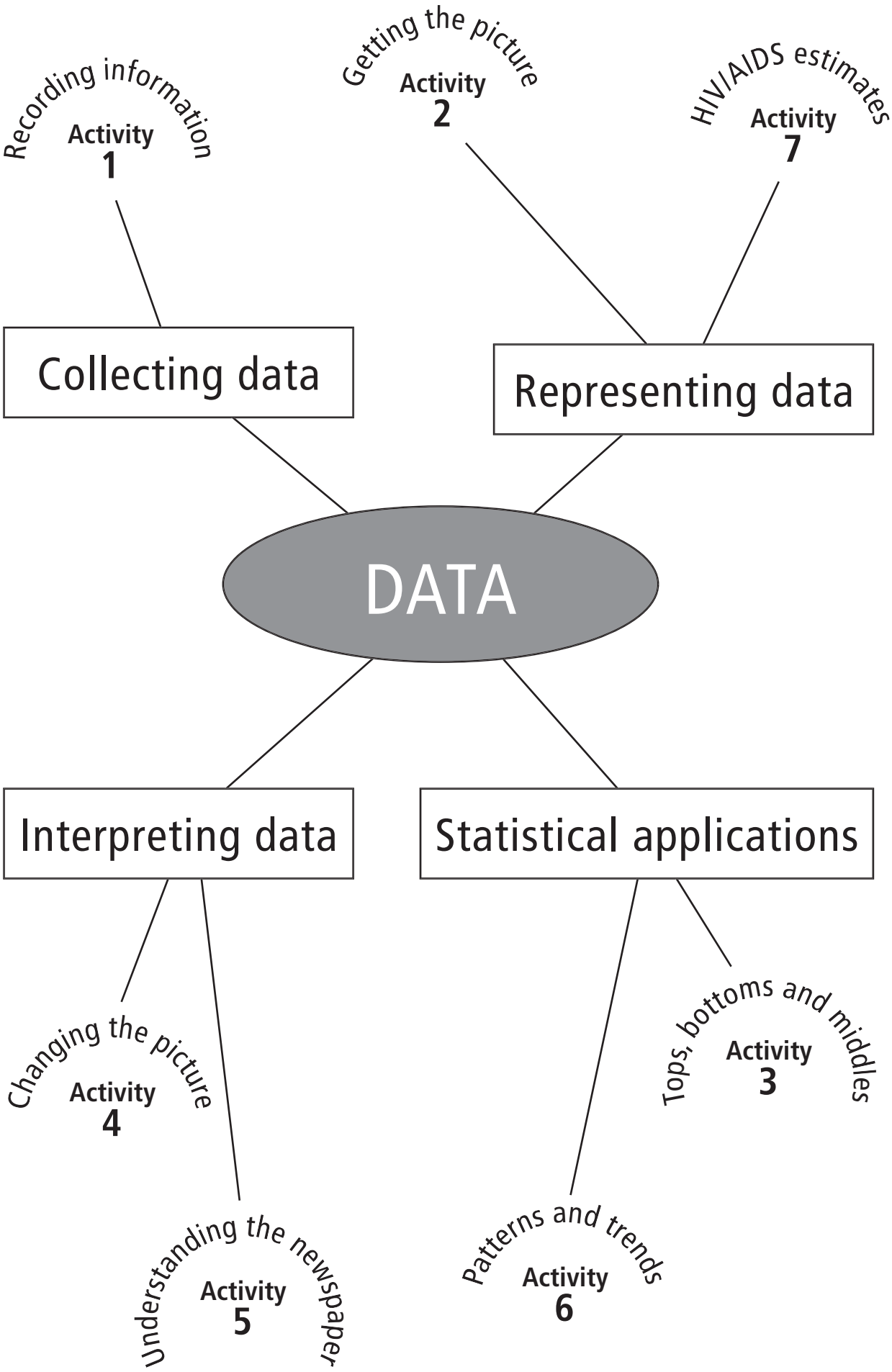
Mathematical Literacy

NQF LEVEL 2



Unit 1
Data

Generic Business Administration


Recording information
Activity 1
Getting the picture
Activity 2
HIV/AIDS estimates
Activity 7
Collecting data
Representing data
DATA
Interpreting data
Statistical applications
Changing the picture
Activity 4
Tops, bottoms and middles
Activity 3
Understanding the newspaper
Activity 5
Patterns and trends
Activity 6

Overview

In this unit students develop a variety of mathematical skills including:

- Data collection and organisation, using a questionnaire about grocery stores, brand names and cell-phones.
- Graphical representation of the data collected using bar graphs and pie charts.
- Understanding cumulative data, as well as developing and interpreting frequency tables.
- Analysis of data to answer questions, using graphs and tables.
- Reading and interpreting figures, in the context of a newspaper report concerning skills development.
- Calculating the various measures of central tendency and percentages in the context of workers and wages.
- Working with range and spread of data.
- Identifying distortion and manipulation of information.
- Identifying patterns and trends, through working with scatterplots and broken line graphs.
- Making simple predictions and comparisons.

Activity 1: Recording information
This is a preliminary activity designed to give students an opportunity to collect a small amount of raw data and transcribe this information onto a frequency table. Students collect information about grocery stores, brand names and cellphones, using a questionnaire. They then construct frequency tables and make various comparisons.

Activity 2: Getting the picture
In this activity students represent data in three different ways — on a single bar graph, a double bar graph and a pie chart. They work with frequency tables similar to the ones they completed in the previous activity.

Activity 3: Tops, bottoms and middles
This activity introduces the ideas of central tendency, range and spread. Students calculate the mean, median and mode for ungrouped data. They organise this data into groups and construct a grouped frequency table and histogram of the raw data. They do this to compare the wage structures of two companies where there are differences in variance and spread.

Activity 4: Changing the picture
In this activity, students read information from existing graphs and identify ways in which these graphs can be used to manipulate and distort the information. They get further practice in drawing their own graphs and change the scale on the vertical axis to observe the effects.

Unit outcomes

The following Unit Standards, Specific Outcomes and Assessment Criteria are addressed by this unit:

Work with a range of patterns and functions and solve problems (9007)

- Give opinions on the implications of the modelled data for the required purpose. (SO1)
  - Verbal (written or oral) explanation of findings is based on the representation of data. (AC1)
  - Trends, group profiles and attitudes are justified. (AC2)
- Appropriate information is extracted from representations in order to answer questions. (AC3)
  - Apply various techniques to organise and represent data. (SO2)
  - Questions that can be dealt with through statistical methods are identified. (AC1)
  - Existing tables are understood correctly through proper application of row and column headings. (AC2)
  - Raw data and tables are correctly used.
  - Data are recorded and organised to solve problems correctly. (AC4)
  - Calculations of statistics are correct (AC5)
  - Appropriate statistics are used to answer questions. (AC6)
  - Graphical representations are correct
  - and consistent. (AC7)

Activity 5: Understanding the newspaper
In this activity students examine information from a table given in a newspaper article. They are required to read values in rows and columns and understand the relationship between them, including those with three variables. They are working with much larger numbers in this exercise and accurate calculator work is needed. The context is skills development projects in the provinces organised by the Department of Labour.

Activity 6: Patterns and trends
In this activity students identify patterns and trends using scatter plots, broken line graphs and bar graphs. They read values from given graphs and use them to do calculations. They make simple predictions by identifying trends.

Activity 7: HIV/AIDS estimates
In this activity students are given two world maps with estimated figures for HIV/AIDS. They read figures, calculate averages, make comparisons, tabulate data and represent it on a bar graph.

14085	**Apply basic knowledge of statistics in order to investigate life- and work-related problems**								
		ACTIVITY	1	2	3	4	5	6	7
SO1	Give opinions on the implications of the modelled data for the required purpose								
	AC1	Verbal (written or oral) explanation of findings is based on the representation of the data.	✓	✓		✓	✓	✓	✓
	AC2	Trends, group profiles and attitudes are justified.				✓	✓	✓	
	AC3	Appropriate information is extracted from representations in order to answer questions.	✓	✓		✓	✓	✓	✓
SO2	Apply various techniques to organise and represent data								
	AC1	Questions about sets of data that can be dealt with through statistical methods are identified correctly.	✓	✓	✓				✓
	AC2	Existing tables are understood correctly through a proper application of row and column headings.	✓	✓	✓	✓	✓		✓
	AC3	Raw data or statistics in the body of tables are used correctly.	✓	✓	✓				✓
	AC4	Effective methods to record and organise data are used to solve problems.			✓				
	AC5	Calculations of statistics are correct.			✓	✓	✓		✓
	AC6	Appropriate statistics are used to answer questions.				✓		✓	
	AC7	Scales used in graphical representations and tables are consistent with the data, are correct, clear and appropriate to the situation and target audience.		✓		✓			✓

Activity 1 — Recording information

ABOUT THIS ACTIVITY

This is a preliminary activity designed to give students an opportunity to collect a small amount of raw data and transcribe this information onto a frequency table. Students collect information about grocery stores, brand names and cellphones, using a questionnaire. They then construct frequency tables and make various comparisons. This activity addresses the following Specific Outcomes and Assessment Criteria: SO1 – AC1, 3; SO2 – AC1, 2, 3.

MANAGING THIS ACTIVITY

Introduce this section by reminding students that statistics is the section of mathematics that uses numerical methods to record, represent and analyse a range of information from a variety of sectors. We can examine weather patterns and predict water shortages, we can survey opinions, we can test medicines and record success rate, we can predict market trends and analyse inflation rates. Statistics are often used as the basis of argument and decision making, and the analysis of information is an important part of everyday life.

We use the term data to mean 'information' and this data can be collected in different ways, for example surveys, interviews or questionnaires. The data is then represented on different types of graphs and tables as can be seen every day in newspapers, in magazines and on the internet. By studying patterns and making calculations, we are able to draw conclusions, make decisions and predict trends. We need to be aware, however, that information can be unreliable and graphs can be distorted, resulting in inaccurate interpretations.

To help count large numbers of data items, the tally counting method is often used. This method uses a vertical line for each data item and a slash across four lines for every fifth item (IIII). The groups of five can then quickly and easily be counted. Students should know that the word frequency means how many times and that we often use the letter F to represent frequency.

This introduction could be done as a question and answer session to assess how much knowledge the students already have, or as a teacher led discussion. Each student should then be given a copy of Handout 1 and Worksheet 1. The questionnaire could be completed by members of the class, but it would be more interesting if it were filled in at home by a greater range of people. Students should record data using ticks in the appropriate boxes.

Each student will have a different set of data, so the completed tables that follow are not definitive. A set of possible answers has been used in order to illustrate the underlying principles.

1.1

Comparing the numbers of people who use each store most often		
Grocery Store	Tally	Frequency
Pick 'n Pay	~~IIII~~ III	8
O.K.	IIII	4
Spar	~~IIII~~ IIII	9
Woolworths	~~IIII~~	5
7-11	II	2
Other	II	2
Total		30

Spar and Pick 'n Pay were used most often by this group of people, while 7-11 and other stores were used least. The total frequency should be 30 in each case.

1.2

Comparing the most frequently used store of males and females in 25+ age group						
Store	Pick 'n Pay	O.K.	Spar	W'worths	7-11	Other
Male	2	1	0	1	0	0
Female	5	3	1	2	3	0

The store used most often by women in this sample is Pick 'n Pay. Other stores were used the least by both men and women.

1.3

Comparing clothing brand name preference between under 25s and over 25s							
Brand	Red	Roxy	Billabong	Quicksilver	Nike	Other	Total
≤ 25years	3	9	2	0	2	5	21
> 25years	0	0	0	2	4	3	9

In this sample, 21 of the 30 people were under 25 and Roxy was the favourite brand in this group. The over 25s in this sample did not favour Red, Roxy or Billabong. Their favourite brand was Nike. Other brands scored well in both age groups. Notice that the sum of the totals should be 30.

1.4

Comparing brand name preferences between males and females							
Brand	Red	Roxy	Billabong	Quicksilver	Nike	Other	Total
Males	1	0	1	2	5	4	13
Females	2	9	1	0	1	4	17

In this sample, 17 were female and 13 were male. Roxy was the favourite for women and Nike for the men. The women did not favour Quicksilver and the men did not favour Roxy.

1.5

Network provider				
Type	Cell C	Vodacom	MTN	Total
Male	4	6	3	13
Female	6	5	6	17
≤ 25 years	7	8	6	21
> 25 years	3	3	3	9

In this sample, Vodacom was the most popular cellphone type by a small margin. However, there was no clear favourite and the spread was fairly even across all types and age groups.

1.6 A variety of answers would be expected. This is an opportunity to raise awareness in students that peer pressure and advertising is very good for the profits of the manufacturer but not always so good for the consumer, who could perhaps get an equally good article, without a brand name, for a much better price. The discussion could also touch on the idea that supporting the local market is good for local economic development, but that many popular brand names are not local.

Activity 1 — Recording information

Using the questionnaire printed on Handout 1, you first need to collect information from 30 different people on which grocery store they use most often, which clothing brand name is their favourite and which network provider they like best. You also need to record whether they are male or female and whether they are more than 25 years old, or from 0 - 25 years old. Use ticks in the correct boxes and then use the given table to summarise your data.

Now complete the following tables using the data you collected. In the space below the tables write two conclusions you can make from the table.

1.1

Comparing the numbers of people who use each store most often		
Grocery Store	Tally	Frequency (F)
Pick 'n Pay		
O.K.		
Spar		
Woolworths		
7-11		
Other		
Total		

1.2

Comparing the most frequently used store of males and females in 25+ age group						
Store	Pick 'n Pay	O.K.	Spar	W'worths	7-11	Other
Male						
Female						

1.3

Comparing clothing brand name preferences between the under 25s and the over 25s							
Brand	Red	Roxy	Billabong	Quicksilver	Nike	Other	Total
≤ 25years							
> 25years							

1.4

Comparing clothing brand name preferences between males and females							
Brand	Red	Roxy	Billabong	Quicksilver	Nike	Other	Total
Male							
Female							

1.5

Network provider				
Type	Cell C	Vodacom	MTN	Total
Male				
Female				
≤ 25 years				
> 25 years				

1.6 By using your data and comparing it with the data of at least two other classmates, discuss if there exists a relationship between the types of grocery stores people use, the clothing brand they prefer and the cellular network provider they use.

QUESTIONNAIRE

NAME .....

Please tick the correct block:

1. Gender:
   - Male ☐
   - Female ☐

2. Age:
   - 0 – 25 years ☐
   - 25+ years ☐

3. Which of the following grocery stores do you visit most often?
   - Pick 'n Pay ☐
   - O.K. Bazaars ☐
   - Spar ☐
   - Woolworths ☐
   - 7-11 store ☐
   - Other ☐

4. Which of the following clothing brand names do you like best?
   - Red ☐
   - Roxy ☐
   - Billabong ☐
   - Nike ☐
   - Quicksilver ☐
   - Other ☐

5. Which cell phone network provider do you prefer?
   - Cell C ☐
   - Vodacom ☐
   - MTN ☐

Name	Group				Grocery store						Clothing brand						Network provider		
	Male	Female	0-25years	25+ years	Pick'nPay	O.K.	Spar	W'worths	7-11 store	Other	Red	Roxy	Billabong	Nike	Quicksilver	Other	Cell C	Vodacom	MTN
1																			
2																			
3																			
4																			
5																			
6																			
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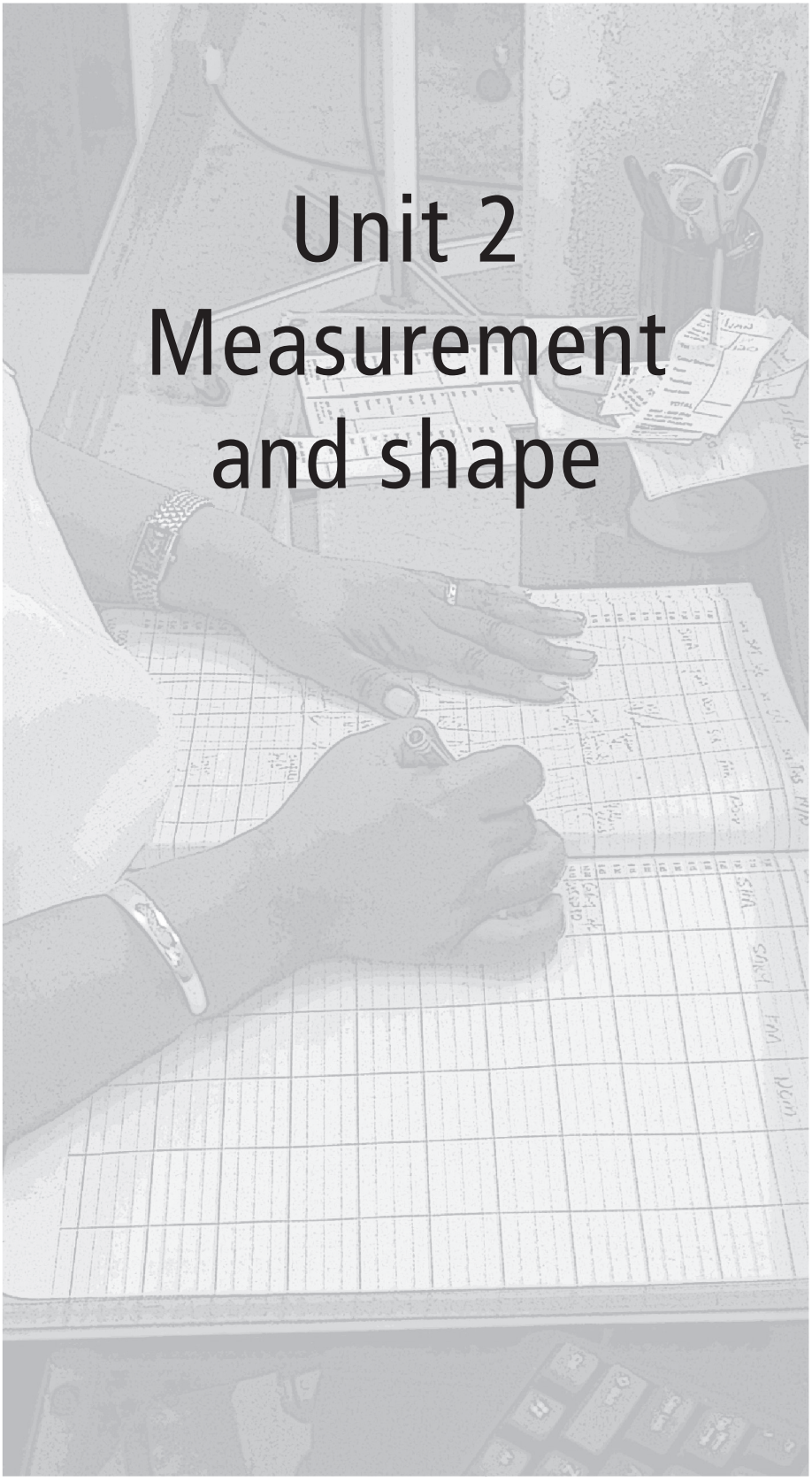
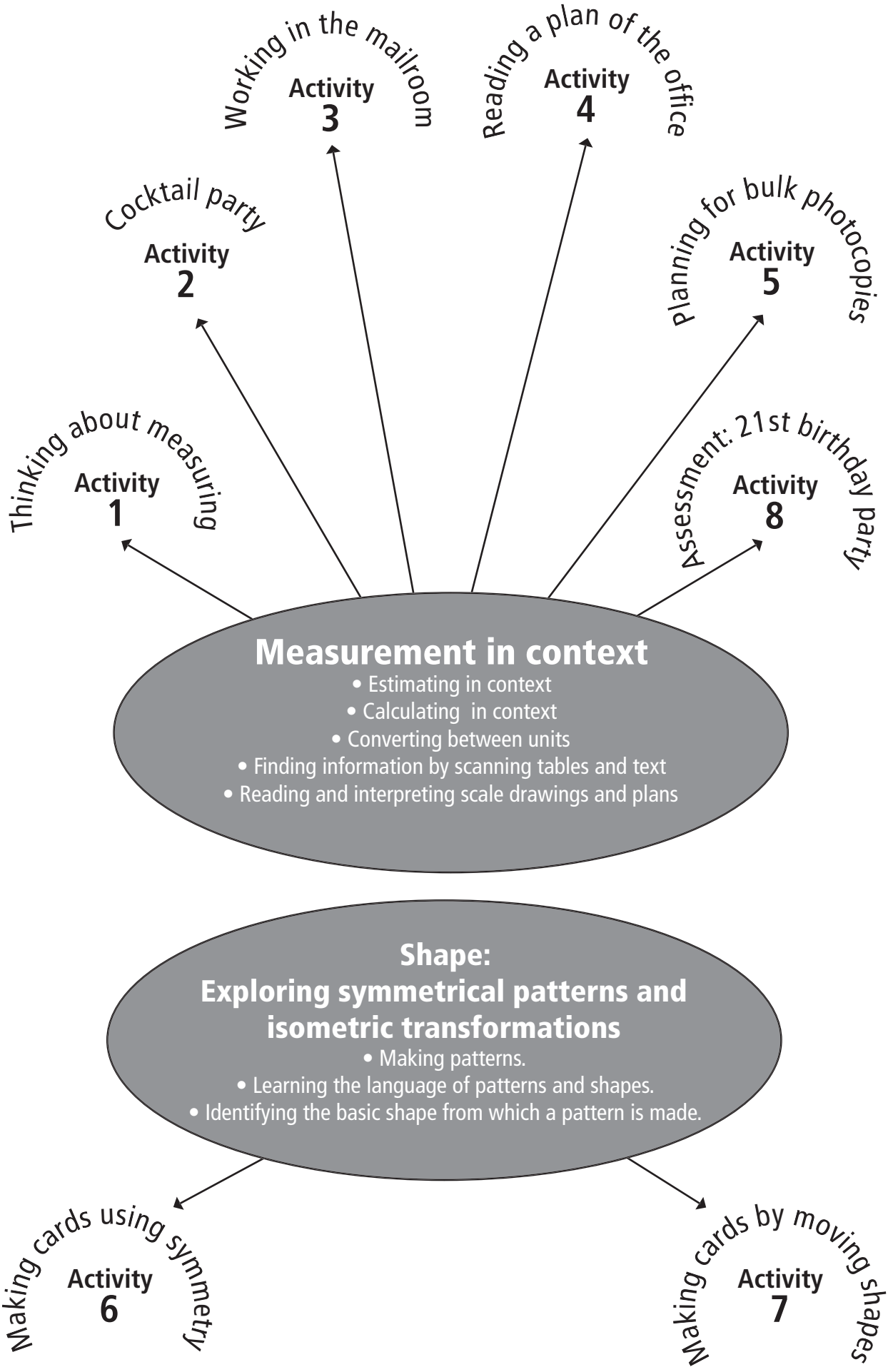
Mathematical Literacy

NQF LEVEL 2



Unit 2 Measurement and shape

Generic Business Administration


Thinking about measuring
Activity 1
Cocktail party
Activity 2
Working in the mailroom
Activity 3
Reading a plan of the office
Activity 4
planning for bulk photocopies
Activity 5
Assessment: 21st birthday party
Activity 8
Measurement in context
• Estimating in context
• Calculating in context
• Converting between units
• Finding information by scanning tables and text
• Reading and interpreting scale drawings and plans
Shape:
Exploring symmetrical patterns and isometric transformations
• Making patterns.
• Learning the language of patterns and shapes.
• Identifying the basic shape from which a pattern is made.
Making cards using symmetry
Activity 6
Making cards by moving shapes
Activity 7

Overview

Unit Standard 12444 has two Specific Outcomes:
- SO 1 focuses on measurement.
- SO 2 focuses on shape – looking particularly at symmetry and how patterns are made by changing the position of a key shape.

This unit uses a number of different contexts to develop students' understanding of measurement, shape and number calculations, including:
- Reading and interpreting information from tables.
- Knowing what units of measurement are appropriate in particular contexts.
- Choosing the appropriate measuring instrument.
- Converting between units of measurement.
- Selecting the most appropriate of measuring for a particular context.
- Thinking about the reasonableness of the measurement reading in a particular context.
- Using the context to determine the level to which one needs to round off.
- Using estimation techniques appropriate to the context.
- Deciding which algorithms to use in calculations based on the context.
- Explaining the choice of algorithm and the procedures used.
- Considering the appropriateness of an answer in terms of the context.
- Using the available technology, in particular electronic calculators and spreadsheet packages, to solve problems
- Calculating effectively, efficiently and correctly.
- Developing models to describe, analyse and make predictions about situations.
- Reading plans.
- Understanding scale in sketches and using ratio to calculate the actual size.
- Working with symmetry and transformation of shapes.

In addition to developing the mathematical skills of students, this unit is also intended to give students an opportunity to develop an understanding of:
- Postal rates at the post office – how to read and interpret them.
- Arranging space in terms of office work flow and staff needs.
- Predicting stationery and other stock requirements to allow for effective planning e.g. for timeous stock purchase and appropriate stock storage.

Mathematical literacy involves using ones knowledge of shape, space, measurement,

Unit outcomes

Measure, estimate and calculate physical quantities and explore, describe and represent geometric relationships in 2-dimensions in different life and workplace contexts. (US 12444)
- Estimate, measure and calculate physical quantities to solve problems in practical situations. (SO1)
  - o Scales on measuring instruments are read correctly (AC1)
  - o Quantities are estimated to a tolerance acceptable in the context in the context of the estimation (AC2)
  - o The appropriate instrument is chosen to measure a particular quantity (AC3)
  - o Calculations are carried out correctly (AC4)
  - o Appropriate units are used in measurement and calculation. (AC5)
  - o Rough sketches are interpreted or used correctly to represent and describe situations. (AC6)
  - o Scales are used correctly in interpreting and describing situations through scale diagrams. (AC7)
- Explore transformations of two-dimensional geometric figures. (SO2)
  - o Properties of symmetrical shapes are recognised and described. (AC1)
  - o The concept of lines of symmetry in 2-dimensional figures is explored using paper folding and reflections in the lines of symmetry. (AC2)
  - o The concept of transformation in terms of reflections, translations and rotations is identified and explained using concrete materials. (AC3)
  - o The descriptions are based on correct application of transformations and other geometrical properties. (AC4)
  - o Designs, based on transformations and other geometrical properties are innovative, and correct geometrically. (AC5)

Demonstrate understanding of rational numbers and irrational numbers and number systems within the context of relevant calculations (US 8982)
- Demonstrate understanding of and carry out calculations with rational numbers and irrational numbers (SO1)
  - o Decisions about the sue of a calculation tool to solve problems are reasonable (AC1)
  - o Computational tools are used effectively and correctly (AC2)
  - o Results of calculations are predicted and verified in terms of the context or problems. (AC3)
  - o Solutions obtained are verified in terms of the context or problems. (AC4)
  - o Computational procedures (algorithms) used are executed correctly on different calculation tools. (AC5)
  - o Calculation procedures (algorithms) used are explained and justified. (AC6)
- Demonstrate knowledge and understanding of the different representations of rational numbers. (SO2)
  - o Common fractions are correctly written as decimal fractions and vice versa. (AC1)
  - o Rational numbers are written in exponential notation. (AC2)
  - o Calculation procedures (algorithms) are explained and justified. (AC4)
- Demonstrate understanding of and use scientific notation. (SO3)
  - o Numbers are rounded off appropriate to contexts. (AC3)

numbers and data to make sense of the world. It also involves using one's knowledge of particular contexts to make sense of and increase one's knowledge of shape, space, measurement, numbers and data.

There are several key issues in measurement and shape, and the activities in this unit attempt to address them in ways that are relevant to business administrators. Business Administration is a qualification that needs to help students to develop a set of general skills as well as the flexibility to apply these in a range of contexts. In this unit we provide students with different situations modelled on real workplace contexts. We help students to understand and think their way into each of the contexts before starting the activities.

This unit develops the skills required to achieve the Assessment Criteria of Unit Standard 12444. It also develops most of the skills required to achieve the Assessment Criteria of Unit Standard 8982.
The unit consists of eight activities of which one is an assessment activity.

Activity 1 – Thinking about measuring
In this activity students think about what units of measurement and what measuring instruments they would use for measuring different things. The activity also revises the basic units of measurement and gives students ideas about estimation.

Activity 2 – Organising a cocktail party
In this activity students focus on measuring capacity and budgeting around these measurements. The activity uses the context of preparing for a cocktail party. This requires students to think about capacities of various containers in ways that enable them to estimate the appropriate amounts and to calculate what amounts to order. Students are asked to do calculations based on tables of data and to consider their answers in terms of this specific context.

Activity 3 – Working in a mailroom
In this activity students work with mass, lengths and costs. They are required to use the information provided in the postal service tables to "sort"/group mail into appropriate categories and then use the cost tables provided to calculate costs.

Activity 4 – Reading an office plan
In this activity students are asked to interpret the plans of an office. This includes an understanding of scale and ratio. They are asked to reorganise the space to incorporate work stations for three temporary employees. They need to estimate and measure desks and other furniture whilst simultaneously considering other practical aspects of office life such as the need for computers to be near dedicated plugs; the legal requirements about widths of passage ways and the fact that office workers need certain amounts of light and should not be located directly next to photocopy machines.

Activity 5 – Bulk photocopies
In this activity students are asked to convert estimated pages of photocopies into numbers of boxes of photocopy paper. They are then asked to think about the amount of storage space needed for these boxes. Finally, they need to work out about how much shelving they require to temporarily store the photocopied books.

Activity 6 – Making cards using symmetry
In this activity students fold paper and cut out shapes along the fold- line as an introduction to investigating the concept of symmetry.

Activity 7 – Making cards by moving shapes
In this activity students work with transformations of shapes in which the basic shape stays the same but is repeated in different positions and orientations to form complex pictures and patterns.

Activity 8 – Assessment
In this assessment students are asked to plan the beverages for a 21st party. Students will be working with capacity. They need to estimate and round off in ways that are appropriate to the context. They need to look up information about capacity and costs in Handout. They calculate costs and use these calculations to refer back to some of the calculations they did on capacity. As is the case in many of the calculations in this course students will need to think about how the context impacts on estimations and approximations. They will also need to check whether later calculations impact on calculations and decisions.

12444	Measure, estimate and calculate physical quantities and explore, describe and represent geometrical relationships in 2-dimensions in different life or workplace contexts									
Level 2		ACTIVITY	1	2	3	4	5	6	7	8
SO1	Estimate, measure and calculate physical quantities to solve problems in practical situations.									
	AC1	Scales on the measuring instruments are read correctly.			✓					
	AC2	Quantities are estimated to a tolerance acceptable in the context of the estimation.	✓	✓	✓	✓	✓			✓
	AC3	The appropriate instrument is chosen to measure a particular quantity.	✓		✓	✓				
	AC4	Calculations are carried out correctly.	✓	✓	✓		✓			✓
	AC5	Appropriate units are used in measurement and calculation.	✓	✓	✓		✓			✓
	AC6	Rough sketches are interpreted or used correctly to represent and describe situations.			✓	✓	✓	✓		
	AC7	Scales are used correctly in interpreting and describing situations through scale diagrams.			✓	✓				
SO2	Explore transformations of two-dimensional geometric figures.									
	AC1	Properties of symmetrical shapes are recognised and described.						✓		
	AC2	The concept of lines of symmetry in 2-dimensional figures is explored using paper folding and reflections in the lines of symmetry.						✓		
	AC3	The concept of transformation in terms of reflections, translations and rotations is identified and explained using concrete materials.							✓	
	AC4	The descriptions are based on correct application of transformations and other geometrical properties.							✓	
	AC5	Designs, based on transformations and other geometrical properties are innovative, and correct geometrically.						✓	✓	

	Demonstrate understanding of rational numbers and irrational numbers and number systems within the context of relevant calculations.									
8982	Calculations used in activities for US 12444									
Level 2		ACTIVITY	1	2	3	4	5	6	7	8
SO1	Demonstrate understanding of and carry out calculations with rational numbers and irrational numbers.									
	AC1	Decisions about the use of a calculation tool to solve problems			✓	✓	✓			
	AC2	Computational tools are used effectively and correctly	✓	✓	✓	✓	✓			✓
	AC3	Results of calculations are predicted and by estimation	✓	✓	✓	✓	✓			✓
	AC4	Solutions are verified in terms of the contexts of problems	✓	✓	✓	✓	✓			✓
	AC5	Computational procedures (algorithms) are executed correctly on different calculation tools	✓	✓	✓	✓	✓			✓
	AC6	Calculation procedures (algorithms) used are explained and justified	✓	✓	✓	✓	✓			✓
SO2	Demonstrate knowledge and understanding of different representations of rational numbers									
	AC1	Common fractions are correctly written as decimal fractions and vice-versa			✓					
	AC2	Rational numbers are written in exponential notation				✓				
	AC3	The appropriate equivalent representation of rational numbers is used in calculations								
	AC4	Calculation procedures (algorithms) are explained and justified	✓	✓	✓	✓				✓
SO3	Demonstrate understanding of and use scientific notation									
	AC1	Very large and very small numbers are converted to scientific notion correctly								
	AC2	Scientific notation is interpreted and used appropriately in real life contexts								
	AC3	Numbers are rounded off appropriate to contexts	✓	✓	✓	✓	✓			✓

Activity 1 — Thinking about measuring

ABOUT THIS ACTIVITY

Before students begin working with measuring and calculating we ask students to first make sense of the system of measurement that we use. We ask them to think about which units and instruments are appropriate for measuring in which contexts.

In this millennium, it is perhaps more important that people develop the skills of looking up information, than that they learn information. In this first activity students are asked to look up and make sense of information provided in different tables. It may initially be difficult for students to make sense of how some of the tables work, but once they have developed the skill of decoding the tables it will be much easier for them to do on other occasions.

This activity prepares students to meet the following outcomes of Unit Standard 12444:

- SO 1: Estimate, measure, and calculate physical quantities to solve problems in practical situations: AC 2, 3, 4, 5

This activity also prepares students to meet the following outcomes of Unit Standard 8982:

- SO 1: Demonstrate understanding of and carry out calculations with rational numbers and irrational numbers: AC 2, 3, 4, 5, 6
- SO 2: Demonstrate knowledge and understanding of the different representations of rational numbers: AC 4
- SO 3: Demonstrate understanding of and use scientific notation: AC 3

MANAGING THIS ACTIVITY

As with all the measuring activities there are things that you can do to make the questions more real or more meaningful to the students with whom you work. In this activity we provide some examples of each kind of measurement (mass, capacity, length and distance, area, volume, speed, temperature etc.) The questions may make more sense to students if you adapt them to objects that they can see or to contexts with which your particular group of students are familiar. For instance in question 1.5.2, ask about the width of the road outside their college – then they can go outside and have a look, also ask them about measuring objects that they can see in the classroom etc.

Many office diaries (as well as most encyclopedias) have information on the metric system and how units are related. Students need to have the experience of working with these tables, but it is also important for them to know where to find these tables. If possible, find similar tables in diaries for your students. These need not be current diaries – diaries from previous years will also have tables in them. But the experience of looking up in the information in the diary is an important skill for them to take into the world of work.

These questions require students to make sense of the table: there are two skills involved:

- o students need to read the table,
- o they also need to interpret the information provided.

1 The first 3 questions are fairly straightforward. Students should be able to complete them on their own or by working in pairs.

1.1 These are some possible answers to the questions. Students may also provide other examples.

Distance and length	Mass	Capacity	Volume	Area
Length of room Height of person Distance between towns Distance to a place you want to go Length of string, piping, fencing, material, wooden planks etc when making things	Mass of a person Mass of ingredients when cooking Mass of a parcel for posting.	Amount of liquid when cooking Amount of liquid medicine Capacity of a car engine Amounts of liquids when mixing substances e.g. insecticides for garden	Volume of a box, cupboard or storage room when needing to pack or store things	Surface area of walls for painting or tiling or wall papering Surface area of floors for carpeting or tiling

1.2 Before starting with this activity check that students understand what the term "units" means in the context of measurement. As with so many words in mathematics "unit' can mean different things outside the maths classroom. Students may think of kitchen units i.e. cupboards; unit trusts or they may also think of tens and units. Students do not need to define units, rather ask them to brainstorm any units of measurement that they can think of.

Distance and length	Mass	Capacity	Volume	Area
Centimetre Yard Kilometre Inch	Kilogram Gram Tonne Pound	Litre Kilolitre Pint Millilitre Gallon	Cubic metre	Square centimetre

1.3 Students may still find themselves in situations where imperial measures are used. One such situation is talking with older people. Another is if they work for a business that has international clients or suppliers.

1.3.1 Metric system: litre, kilogram, centimetre, kilolitre, millilitre, kilometre, gram, tonne, cubic metre, square centimetre.

1.3.2 Imperial: ounce, pint, yard, inch, gallon, pound.

1.4 It is important that students are able to estimate and have some sense of size. Regular practice with estimating measurements can improve ones estimation skills. It is better for students to do a little bit of estimation regularly than do a lot all at once. It is also useful for student's to have a set of objects or practices of known length against which they can compare other lengths. For example the following are all about 1 metre:

- a standard pace
- the length from ones shoulder to the opposite finger tip of an outstretched arm
- the height of an average window.
- ½ the height of an average door.

These are useful for students to refer to when considering and estimating the lengths and heights of other things. Let students have regular practice in using these and other common referents to estimate size.

You many need to explain the following terms to students:

- body measures
- informal referents i.e. everyday things against which you can compare the measurements of other objects

1.4.1 Pace it out.

1.4.2 The length from ones shoulder to the opposite finger tip of an outstretched arm.

1.4.3 For many people the width of their closed flat hand is about 10 cm at some point.

1.5 If students are not able to think of the appropriate units then you could supply a range of units. If measuring length then include some units that can be used for measuring length or distance but are not suitable for the object or distance given, as well as units which cannot be used for measuring the object e.g. litres or square centimetres.

To know what units to use, students first need to know whether they are measuring length/distance; mass; capacity; area; volume; time or speed. Then they need to think about the size of what they are measuring to choose whether to use, for example, millilitres, litres or kilolitres. However, there some social conventions that break the rule e.g. building materials are usually measured in millimeters (glass, nails, etc) while body measurements are usually given in centimetres (clothing, body lengths, etc).

1.5.1 Kilometres

1.5.2 Metres

1.5.3 Centimetres

1.5.4 Square centimetres

1.5.5 Kilograms

1.5.6 Kilograms

1.5.7 Grams

1.5.8 Litres

1.5.9 Millilitres

1.5.10 Millimetres - it is conventional to measure glass and other building materials in millimeters. Here it is not the size that determines the unit of measurement.

1.5.11 Millimetres – paper sizes are given in millimeters by convention. Here too it is not the size that determines the unit of measurement e.g. an A4 size of paper is 210 mm × 297 mm, an A0 size of paper is 841 mm × 1189 mm.

1.5.12 Millilitres

1.5.13 Litres

1.5.14 Kilolitres

1.5.15 Square metres

1.5.16 Millimetres

1.5.17 Millimetres

1.6.1 Thermometer

1.6.2 Bathroom scale

1.6.3 Measuring tape

1.6.4 Measuring jug or cups

1.6.5 Kitchen scale, measuring jug or measuring cups

1.6.6 Measuring spoon

1.6.7 Tape measure

1.6.8 Tape measure

1.6.9 Odometer

1.6.10 Speedometer

1.7 The handout provides tables in some of the forms that are commonly found in commercial diaries. Students may take some time to make sense of the tables. It is important that they learn this skill because in the world of work they will not always find information presented in forms that are exactly suitable for their particular purpose at the time. Some information is given in more than one table. They can then choose the easiest form of table to read, or use the simpler of the two tables to help them decode the less obvious one.

1.7.1 Metre

1.7.2 Kilograms

1.7.3 Seconds

Most FET students do not have any sense of imperial measures. It is useful for them to have some sense of the size of some imperial measures relative to metric and other everyday units of measurement. It is important to stress that these are only approximations. Let them compare the approximations with the accurate tables to see how rough the approximations are.

1.8.1 Inch

1.8.2 Metre

1.8.3 Mile

1.9.1 Kilogram

1.9.2 Ounce

1.10 Litre

1.11 About 12. One ruler length is about 1 foot and there are 3 feet in a yard, so there are about 12 ruler lengths in a yard.

1.12 Less than 4 metres.

1.13 About 3 litres.

1.14 About 2 kilograms.

1.15 About 25 cm.

1.16.1 A six inch nail is a152mm nail.

1.16.2 2 pints of milk is 1 137 millilitres or about 1 litre.

1.16.3 3 pounds of butter is 1 361 grams or about 1,4 kg.

1.16.4 5 yards is 4 572 mm or about 4,5 m.

1.16.5 80 miles per hour is 128 km per hour.

1.16.6 105°F is 40°C.

1.16.7 9 × 12 carpet is 9 feet by 12 feet. This is almost the same size as 3 m × 4 m.

1.17 Most people will find the table on page 1 of the Handout easier to read, but the other table provides much more information. Students need to understand that they can start with the number in the middle. If they are converting inches to centimetres, then they can say 1 inch equals 2,540 cm by reading off the centimetres on the right hand side. If they are converting centimetres to inches then they can say 1 cm = 0,394 inches by reading the inches off the left hand column.

Check also that students know how to convert between related metric units e.g. centimetres to metres etc.

Activity 1 — Thinking about measuring

Before we look at how measuring can help us at work and at home, let's first think what we already know about measuring and measurements.

1.1 Copy the table below. Fill in two examples of when you use each kind of measurement e.g. you measure mass when standing on a bathroom scale.

Distance and length	Mass	Capacity	Volume	Area

1.2 In the box below are different units of measurement. Copy the table in 1.1 again and fill in the units of measurement in the correct columns.

litre (*l*)	kilogram (kg)	centimetre (cm)	ounce (oz)
kilolitre (K*l*)	pint (pt.)	yard (yd)	millilitre (m*l*)
kilometre (km)	gram (g)	tonne (T)	inch (in)
pound (*lb*)	gallon (gal)	cubic metre (cm^3)	square centimetre (cm^2)

1.3 Some of the units above are part of the metric system. Some are part of the imperial system which we used in South Africa before 1960.

1.3.1 Which units above are part of the metric system?

1.3.2 Which units above are part of the imperial system?

1.4 Often we only need to have a rough idea of the size of things. Whenever we measure it is important to estimate first. Talk to a partner and then say how you would estimate the following using informal body measures:

1.4.1 The length of the room

1.4.2 The length of a piece of material

1.4.3 The length of a piece of paper

1.5 What units would you use if you wanted to measure:

1.5.1 The distance between Cape Town and Worcestor?

1.5.2 The width of Broad Road in Wynberg?

1.5.3 The length and width of your desk?

1.5.4 The size of your kitchen table if you wanted to make a table cloth?

1.5.5 How heavy a cow is?

1.5.6 How heavy a newborn baby is?

1.5.7 How much a packet of herbs or spices contains: for example cardomon or bay leaves?

1.5.8 The amount of water a bucket can hold?

1.5.9 The amount of water a milk jug can hold?

1.5.10 The length and breadth of your bedroom window if you want to replace a broken pane of glass?

1.5.11 The length and width of a piece of paper

1.5.12 How much a kitchen cup or wine glass can hold?

1.5.13 How much water it takes to fill a bath?

1.5.14 How much water your family uses in a month?

1.5.15 What the size of a property is that you live on?

1.5.16 The size of a nail or a drill bit?

1.5.17 The size of a button hole?

1.6 What instrument would you use to measure:
1.6.1 Temperature?
1.6.2 Your own mass?
1.6.3 A piece of glass for a picture?
1.6.4 The amount of milk for a cake you are making?
1.6.5 Flour for a recipe you are making?
1.6.6 The amount of medicine to give to a young child?
1.6.7 The area of your bedroom walls if you wanted to paint them?
1.6.8 The distance around someone's waist if you were making her a pair of trousers?
1.6.9 The distance you have travelled in a car?
1.6.10 The speed of the car you are travelling in?

1.7 Handout 1 gives lists of measurements.
1.7.1 Which table shows the metric system that we use? Write down what the standard units of measurement are for:
1.7.2 Length
1.7.3 Mass
1.7.4 Time

The table with the pictures shows handy ways to remember the approximate sizes of Imperial measurements. Use this table to answer the following questions:

1.8 Which is longer:
1.8.1 An inch or a centimetre?
1.8.2 A yard or a metre?
1.8.3 A mile or a kilometer?

1.9 Which is heavier:
1.9.1 A pound or a kilogram?
1.9.2 An ounce or a gram?

1.10 Which is more: a litre or a pint?
1.11 About how many ruler lengths is 4 yards?
1.12 Is 4 yards more or less than 4 metres?
1.13 About how many litres is 6 pints?
1.14 About how many kilograms is 4 pounds?
1.15 About how many centimetres is 10 inches?

1.16 Sometimes we need to convert between the metric system and the old imperial system. Use the tables in the handout to say what your granny means when she talks about:
1.16.1 A six inch nail.
1.16.2 2 pints of milk.
1.16.3 3 pounds of butter.
1.16.4 5 yards.
1.16.5 80 miles per hour.
1.16.6 The temperature of a feverish person at 105 degrees Farenheit.
1.16.7 A carpet that is 9 × 12.

Here you must first think what unit granny is using and then think what unit we would convert it into.

1.17 Now talk about the tables. Which table was easiest for you to use? Explain to a partner how you understood the table that that had 3 columns

METRIC AND IMPERIAL MEASURES

Length

1 centimetre (cm)	= 10 mm	= 0,3937 in
1 metre (m)	= 100 cm	= 1,0936 yd
1 kilometre (km)	= 1 000 m	= 0,6214 mile
1 inch (in)		= 2,54 cm
1 yard (yd)	= 36 in	= 0,9144 m
1 mile	= 1 760 yd	= 1,6093 km

Surface or Area

1 sq cm (cm2)	= 100 mm^2	= 0,1550 in^2
1 sq metre (m2)	= 10 000 cm^2	= 1 1960 yd^2
1 sq km (km2)	= 100 ha	= 0,3861 $mile^2$
1 sq in (in2)		= 6,4516 cm^2
1 sq yard (yd2)	= 9 ft^2	= 0,8361 m^2
1 sq mile (mile2)	= 640 acres	= 2,59 km^2

Volume and Capacity

1 cu cm (cm3)		= 0,0610 in^3
1 cu metre (m3)	= 1 000 dm3	= 1,3080 yd^3
1 litre (l)	= 1 dm3	= 0,2200 gal
1 hectolitre (h l)	= 100 l	= 21,997 gal
1 cu inch (in3)		= 16,387 cm^3
1 cu yard (yd3)	= 27 ft3	= 0,7646 m^3
1 pint (pt)	= 20 fl oz	= 0,5683 *l*
1 gallon (gal)	= 8 pt	= 4,546 *l*

Weight

1 gram (g)	= 1 000 mg	= 0,0353 oz
1 kilogram (kg)	= 1 000 g	= 2,2046 *lb*
1 tonne (t)	= 1 000 kg	= 0,9842 ton
1 ounce (oz)	= 437,5 grains	= 28,35 g
1 pound (lb)	= 16 oz	= 0,4536 kg
1 ton	= 20 cwt	= 1,016 t

US Measures

1 US dry pint	= 33,60 in^3	= 0,5506 *l*
1 US liquid pint	= 0,8327 imp pt	= 0,4732 *l*
1 US gallon	= 0,8327 imp gal	= 3,785 *l*
1 short cwt	= 100 *lb*	= 45,359 kg
1 short ton	= 2 000 *lb*	= 907,19 kg

Temperatures

°C	0	5	15	20	30	40	50	60	70	80	90	100
°F	32	41	59	68	86	104	122	140	158	176	194	212

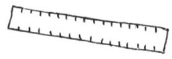
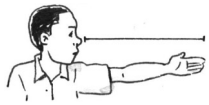
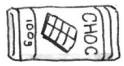
°C = 5/9 (°F–32)
°F = 9/5 °C+32

Speeds

mph	20	30	40	50	60	70	80	90	100
km/h	32	48	64	80	96	112	128	144	160

The Seven Base Units of the SI

Name of unit		*Symbol*
metre	(length)	m
kilogram	(mass)	kg
second	(time)	s
ampère	(electric current)	A
kelvin	(thermodynamic temperature)	K
candela	(luminous intensity)	cd
mole	(amount of substance)	mol

Imperial unit	What it measures	Rough guide to size	Metric unit
feet	length	 1 ruler is about 1 foot	3 feet are about 1 metre 30 cm is about 1 foot
inches (in)	length	top joint of thumb is about an inch	1 inch is about 2 ½ cm
yard (yd)	length	 tip of nose to end of finger	1 yd is slightly less than 1 metre
ounces (oz.)	mass	 100g bar of chocolate is about 1 oz.	100g is a bit less than 4 oz.
pound (lb)	mass	500g of margarine	500g is a bit more than 1 pound
pint (pt.)	volume	500ml of milk	½ a litre is a bit less than 1 pint.

Metric Conversion

LINEAR	**INCHES**		**CENTIMETRES**	**YARDS**		**METRES**	**MILES**		**KILOMETRES**
	0,394	**1**	2,540	1,094	**1**	0,914	0,621	**1**	1,609
	0,787	**2**	5,080	2,187	**2**	1,829	1,243	**2**	3,219
	1,181	**3**	7,620	3,281	**3**	2,743	1,864	**3**	4,828
	1,575	**4**	10,160	4,374	**4**	3,658	2,485	**4**	6,437
	1,968	**5**	12,700	5,468	**5**	4,572	3,107	**5**	8,047
	2,362	**6**	15,240	6,562	**6**	5,486	3,728	**6**	9,656
	2,756	**7**	17,780	7,655	**7**	6,401	4,350	**7**	11,265
	3,149	**8**	20,320	8,749	**8**	7,315	4,971	**8**	12,875
	3,543	**9**	22,860	9,843	**9**	8,230	5,592	**9**	14,484
AREA	**SQ INCHES**		**SQ CENTiMETRES**	**SQ YARDS**		**SQ METRES**	**SQ MILES**		**SQ KILOMETRES**
	0,155	**1**	6,452	1,196	**1**	0,838	0,386	**1**	2,590
	0,310	**2**	12,903	2.392	**2**	1,672	0,772	**2**	5,180
	0,465	**3**	19,355	3,588	**3**	2,508	1,158	**3**	7,770
	0,620	**4**	25,806	4,784	**4**	3,345	1,554	**4**	10,360
	0,775	**5**	32,258	5,980	**5**	4,181	1,930	**5**	12,950
	0,930	**6**	38,710	7,176	**6**	5,017	2,317	**6**	15.540
	1,085	**7**	45,161	8,372	**7**	5,853	2,703	**7**	18,130
	1,240	**8**	51,613	9,568	**8**	6,689	3,089	**8**	20,720
	1,395	**9**	58,064	10,764	**9**	7,525	3,475	**9**	23,310
VOLUME	**CU YARDS**		**CU METRES**	**PINTS**		**LITRES**	**GALLONS**		**LITRES**
	1,308	**1**	0,765	1,760	**1**	0,568	0,220	**1**	4,546
	2,616	**2**	1,529	3,520	**2**	1,137	0,440	**2**	9,092
	3,924	**3**	2,294	5,279	**3**	1,705	0,660	**3**	13,638
	5,232	**4**	3,058	7,039	**4**	2,273	0,880	**4**	18,184
	6,540	**5**	3,823	8,799	**5**	2,841	1,100	**5**	22,730
	7,848	**6**	4,587	10,559	**6**	3,410	1,320	**6**	27,277
	9,156	**7**	5,352	12,318	**7**	3,978	1,540	**7**	31,823
	10,464	**8**	6,116	14,078	**8**	4,546	1,760	**8**	36,369
	11,772	**9**	6,881	15,838	**9**	5,114	1,980	**9**	40,915
WEIGHT	**OUNCES**		**GRAMS**	**POUNDS**		**KILOGRAMS**	**TONS**		**KILOGRAMS**
	0,035	**1**	28,350	2,205	**1**	0,454	0,001	**1**	1018,0
	0,071	**2**	56,699	4,409	**2**	0,907	0,002	**2**	2032,1
	0,106	**3**	85,049	5,614	**3**	1,361	0,003	**3**	3048,1
	0,141	**4**	113,398	8,818	**4**	1,814	0,004	**4**	4064,2
	0,176	**5**	141,748	11,023	**5**	2,268	0,005	**5**	5080,3
	0,212	**6**	170,097	13,228	**6**	2,722	0,006	**6**	6096,3
	0,247	**7**	198,447	15,432	**7**	3,175	0,007	**7**	7112,3
	0,282	**8**	226,796	17,637	**8**	3,629	0,008	**8**	8128,4
	0,317	**9**	255,146	19,842	**9**	4,082	0,009	**9**	9144,4

The key figure printed in **BOLD** in the centre column can be read as either the metric or the British measure, thus 1 metre = 1,09 yards or 1 yard = 0,91 metre

Activity 2 — Organising a cocktail party

ABOUT THIS ACTIVITY

In this activity students are asked to plan the beverages for a cocktail party. Students will be working with capacity and need to estimate and round off in ways that are appropriate to the context. They need to look up information about capacity and costs in reference tables. They calculate costs and use these calculations to refer back to some of the calculations they did on capacity. In this way the activity aims to alert students both to the need for context bound approximations, and also to the fact that some calculations impact on other calculations and decisions.
In this activity students first need to predict what proportion of people will drink each type of beverage. This differs from social group to social group. So in this activity we provide the ratio's within which students should work. Then students calculate how much of each beverage people are likely to drink. They need to convert glasses to millilitres and then factor in the number of people who prefer that beverage. Finally students need to decide what sort of quantities to buy. Different beverages are sold in different quantities. Full bottles, kegs and boxes of juice can be returned, but students should also remember that since it is only a cocktail party and not a wedding or 21st birthday party, they can round down rather than rounding up quantities.

This activity prepares students to meet the following outcomes of Unit Standard 12444:
- SO1: Estimate, measure, and calculate physical quantities to solve problems in practical situations: AC2, 4, 5

This activity also prepares students to meet the following outcomes of Unit Standard 8982:
- SO1: Demonstrate understanding of and carry out calculations with rational numbers and irrational numbers: AC2, 3, 4, 5, 6
- SO2: Demonstrate knowledge and understanding of the different representations of rational numbers : AC4
- SO3: Demonstrate understanding of and use scientific notation: AC3

MANAGING THIS ACTIVITY

If you have students that object for moral, religious reasons or other reasons to working with an activity that involves alcohol, you can change the activity to include bottles of non-alcoholic "champagne" instead of wine, and carbonated colas instead of beer.

Discuss with students the fact that with different gatherings and groups of people the ratios of beverage preference will differ. You can find out what the ratio preferences in the class are and base the activity on these. The ratios in the activity are simple so alternative ratio's may make the calculations more interesting and more meaningful but also more complex.

You can also let students think about what sorts of drinks different groups of people may like. For example they could talk about what drinks they think they would need to serve at:
- the opening of an art exhibition.
- the end of season party at a rugby club.
- a graduation ceremony for MBA students at a business school.
- A graduation party for conservation field workers.
- A 100th birthday celebration in a retirement village.
- A party to mark the end of a traditional initiation ceremony.

The most difficult part of the worksheet is calculating the ratios of people who prefer different kinds of wine. It will help you to assess what students understand about fractions and ratios. You may find that you need to do more work with students on fractions. There are two aspects to this:
- Firstly, do they understand the concepts involved in taking fractions of fractions
- Secondly ,do they know how to do the calculations.

Here we show in pictures a way for students to understand the fractions of fractions used in this activity.
We can think of

$\frac{1}{5}$ as 1 person out of every 5 people.

$\frac{2}{5}$ as 2 people out of every 5 people.

So we can show the fractions or ratios of what this group likes to drink in the following way:

prefers juice | prefers beer | prefers beer | prefers wine | prefers wine

When we want to think about the sub-divisions of who prefers what kind of wine we can show and think about it as follows:

2 out of every 5 people in this group prefer to drink wine. Of these two people ½ like red wine and ½ like white wine. This means that 1 out of every 5 people in this group prefer to drink red wine and 1 out of every 5 people in this groups prefer to drink white wine.

prefers juice | prefers beer | prefers beer | prefers red wine | prefers white wine

We can also show this more symbolically

prefers juice	prefers beer	prefers beer	prefers red wine	prefers white wine

If we then sub-divide the white wine drinkers into the types of white wine they prefer we can show it diagrammatically like this:

prefers juice	prefers beer	prefers beer	prefers red wine	prefers white wine	
					Prefers semi sweet white wine
					Prefers dry white wine

If you then count the small blocks you can see that 2 out of 25 small blocks show preference for semi-sweet white wine, and 3 out of 25 blocks show preference for dry white wine.

This should help students to understand the following:

- Half of the two fifths of wine drinkers will drink red wine i.e. one fifth.
- Half of the two fifths of wine drinkers will drink white wine i.e. one fifth. Of this one fifth, three fifths will drink dry white and two fifths will drink semi-sweet i.e. 3 twenty-fifths of the total will drink dry white and 2 twenty-fifths of the total will drink semi-sweet white wine.

2.1 $\frac{1}{5}$ of 250 people = $\frac{1}{5} \times \frac{250}{1} = \frac{250}{5}$ = 50 people expected to drink juice.

2.1.1 $\frac{2}{5}$ of 250 people = $\frac{2}{5} \times \frac{250}{1} = \frac{500}{5}$ = 100 people expected to drink beer.

2.1.2 1/5 of 250 people is 50 people that are expected to drink red wine.

2.1.3 3/25 of the people are expected to drink dry white wine: We can work this out as follows:

$\frac{3}{25}$ of 250 people = $\frac{3}{25} \times \frac{250}{1} = \frac{3}{1} \times \frac{250}{25} = 3 \times 10 = 30$ people expected to drink dry white wine.

Some students may find it easier to calculate as follows:

50 people are expected to drink white wine. $\frac{3}{5}$ of these will drink dry white wine =

$\frac{3}{5} \times \frac{50}{1} = 3 \times 10 = 30$ people.

2.1.4 2/25 of the 250 people are expected to drink semi-sweet white wine. We can work this out as follows:

$\frac{2}{25}$ of 250 people = $\frac{2}{25} \times \frac{250}{1} = \frac{2}{1} \times \frac{250}{25} = 2 \times 10 = 20$ people expected to drink semi-sweet wine.

Some students may find it easier to calculate as follows:

50 people are expected to drink white wine. $\frac{2}{5}$ of these will drink semi-sweet white wine = $\frac{2}{5} \times \frac{50}{1} = 2 \times 10 = 20$ people.

2.2.1 50 people each drink 2 glasses of 375 ml. $50 \times 2 \times 375 = 100 \times 375$ ml = 37 500 ml. A total of 37 500 ml expected to be consumed. 37, 5 litres of fruit juice needed.

The assumption here is that people are using the beer glasses for fruit juice. You can ask students to do another calculation using wine glasses.

2.2.2 50 people each drink 3 glasses of 375 ml. $50 \times 3 \times 375 =$ 56 250 ml.

A total of 56 250 ml or 56,25 litres of beer expected to be consumed.

2.2.3 Red wine: 50 people each drink 2 glasses of 185 ml. $50 \times 2 \times 185$ml = 100×185 ml = 18 500 ml. A total of 18 500 ml or 18,5 litres of red wine.

2.2.4 Dry white wine: 30 people each drink 2 glasses of 185 ml. $30 \times 2 \times 185$ml = 11 100 ml. A total of 11 100 ml or 11,1 litres of dry white wine.

2.2.5 Semi-sweet white wine: 20 people each drink 2 glasses of 185 ml. $20 \times 2 \times 185$ml = 7 400 ml. A total of 7 400 ml or 7,5 litres of semi-sweet white wine.

2.3 When students get to these calculations remind them that

- The quantities are estimates.
- It is only a cocktail party and not a wedding or a 21st birthday party, so they can round down rather than rounding up, especially with the fruit juice and beer as it is sold in large quantities.

2.3.1 You need about 37,5 litres of fruit juice. Juice comes in 5 litre boxes. 8 boxes give 40 litres. 7 boxes give 35 litres. Since it is only a cocktail party you can round down: 7 boxes should be enough.

2.3.2 56, 25 litres is 3,75 ml short of 60 litres or 2 kegs, and it is 16,25 litres or about 43 beer glasses more than a 50 litre keg can hold. Students may have different answers here. Come back to the answer to this question once students have completed 2.4.2. This is because students will then be able to see that buying two 30 litre kegs is considerably more expensive than buying one 50 litre keg.

2.3.3 You need 18 500 millilitres of red wine. This is 24, 666 bottles of red wine: it is less than two glasses short of 25 bottles. In this example it makes more sense to round up.

2.3.4 You need 11 100 millilitres of dry white wine. This is 14,8 bottles: it is one glass short of 15 bottles so here too it makes sense to round up.

2.3.5 You need 7 400 ml of semi-sweet white wine. Students should be able to see (without calculating) that this is very close to 7 500 ml or 10 bottles of wine. Again, it is less than one glass short of 10 bottles so here too it makes sense to round up.

2.4.1 Cooldrink: Seven 5-litre boxes of juice will cost R244.65.

2.4.2 Beer: Here you should talk to students about the great cost differences between one 50 litre keg and two 30 litre kegs and let them use this to re-consider their answers to question 2.3.2.

50 litre keg costs R620.88 plus R330.00 refundable deposit.

Two 30 litre kegs costs 2 (R452.54) = R905,08 plus R660.00 refundable deposit.

Because of these costs it makes more sense to order the one 50 litre keg.

2.4.3 Red wine: 25 bottles @ R42 will cost R1 050,00.

2.4.4 Dry white wine: 15 bottles @ R30 will cost R450,00.

2.4.5 Semi-sweet white wine: 10 bottles @ R30 will cost R300,00.

2.4.6 TOTAL: R244,65 + R620,88 (plus R330.00 refundable deposit) + R1 050,00 + R450,00 + R300,00 = R 2 645,53 (plus R330.00 refundable deposit). If the refundable deposit is included the cost will be R 2 975,53.

2.5 Students also need to think about hiring glasses and ice.

Activity 2 — Organising a cocktail party

You have been given the task of organising the drinks for a cocktail party to launch a new product. You have invited 350 people. So far, 217 people have said that they are coming. You plan for a total of 250 people.

Your task is to work out what different drinks you need to buy and in what quantities you should buy them.

2.1 Handout 2 gives a breakdown of what proportion of people you can expect to drink each beverage. If you expected 250 people, work out how many people you think will drink each of the following:

2.1.1 Juice

2.1.2 Beer

2.1.3 Red wine

2.1.3 Dry white wine

2.1.4 Semi-sweet white wine and

2.2 Handout 2 also gives details of how much of each beverage you can expect each person to drink. Use this information together with the numbers you calculated in question 2.1 above to calculate how much of each beverage you expect people to drink?

2.2.1 Juice

2.2.2 Beer

2.2.3 Red wine

2.2.4 Dry white wine

2.2.5 Semi-sweet white wine

2.3 Now calculate the total quantities you should order. Remember that any unopened bottles, boxes or kegs can be returned and your money will be refunded.

2.3.1 Juice – How many boxes? What flavours?

2.3.2 Beer – What size keg should you order? How many kegs?

2.3.3 Red wine – How many bottles?

2.3.4 Dry white wine – How many bottles?

2.3.5 Semi-sweet white wine – How many bottles

2.4 Now calculate the cost.

2.4.1 Juice

2.4.2 Beer

2.4.3 Red wine

2.4.4 Dry white wine

2.4.5 Semi-sweet white wine

2.4.6 Total cost

2.5 You have now worked out the quantities and costs of the drinks. What else do you need to plan for to complete the task of organising the liquid refreshments for the cocktail party.

Number of people expected: 250

Expected choices of drink

$\frac{1}{5}$ juice

$\frac{2}{5}$ beer

$\frac{2}{5}$ wine: $\frac{1}{2}$ prefer red wine; $\frac{1}{2}$ prefer white wine.

Of those people who prefer white wine: $\frac{3}{5}$ prefer dry white wine; $\frac{2}{5}$ prefer semi-sweet white wine

Expected quantities to be consumed
Juice – 2 glasses per person: one glass holds 375 ml
Beer – 3 beers per person: one glass holds 375 ml
Wine – 2 glasses per person: one wine glass holds 185 ml

Costs:
Juice – R34,95 for 5 litres.
White wine R30 a bottle. Wine bottles hold 750ml.
Red wine R42 a bottle. Wine bottles hold 750 ml.

Beer

Size of keg/barrel	Cost	Deposit	Cost + deposit
50 litres	R620.88	R330.00	R950.88
30 litres	R452.54	R330.00	R782.54

Activity 3 — Working in the mailroom

ABOUT THIS ACTIVITY

As with most aspects of a company, the main concerns around postage are to have an efficient service and to keep it cost effective. An efficient service keeps customers and clients happy and keeps the business ticking over. It also saves the staff time and trouble. However, if the system is efficient but is too expensive it also puts the business at risk.
The mail room context allows students to practise what they know about reading tables, reading measurements (both in terms of length and mass) and to calculate costs.

We first ask students to think about why one would pre-sort mail rather than just put it all in a bag for the post office to deal with. Then students are asked to think about what sorts of scales/mass metres are suitable to use in a mail room. Finally, students are asked to use measurements (both length and mass) to categorise post.

This activity prepares students to meet the following outcomes of Unit Standard 12444:
- SO1: Estimate, measure, and calculate physical quantities to solve problems in practical situations: AC1, 2, 3, 4, 5, 6, 7

This activity also prepares students to meet the following outcomes of Unit Standard 8982:
- SO1: Demonstrate understanding of and carry out calculations with rational numbers and irrational numbers: AC1, 2, 3, 4, 5, 6
- SO2: Demonstrate knowledge and understanding of the different representations of rational numbers : AC1, 4
- SO3: Demonstrate understanding of and use scientific notation: AC 3

MANAGING THIS ACTIVITY

All prices and information in the Handout of this activity are taken from *Rates 2004/5*: this is a brochure produced by South African Post Office Marketing. Costs will need updating every few years. To get updated brochures, you can write to "The Manager, SA Post Office Marketing, PO Box 10 000, Pretoria, 0001; ask for a copy at your nearest Post Office, or look it up on their website: www.sapo.co.za.

In a real mail room one works with real letters. Looking at and imagining the letters (their size and mass) from a description on a page is not at all the same activity as working with the real objects. As with most measurement activities, aspects of this activity are best done practically. If possible bring in a range of letters and small parcels — these could either be made up for the lesson, or you could ask all students to "collect and save" their unopened mail for a week. The final question gives students practice in measuring lengths, but it would be useful for them to practise measuring the mass of postal items rather than just looking at mass metres in pictures.

It may be useful to clarify some issues around the language of measurement. Talk with students about the fact that what we call a scale in everyday language is technically a mass metre and that the gradations on any measuring instrument can be called the scale. An important part of measuring is understanding that measuring instruments can be calibrated differently: for example a measuring jug can be calibrated in cups/quarter litres, it can be calibrated in 100 ml markings or even 10 ml markings. A skilled measurer always choose a measuring instrument that it calibrated in a manner appropriate to the task and context. This is the basis of question 3.1.2.

Students should be given Worksheet 3 and a copy of Handout 3. Since Worksheet 3 is reasonably straight forward it should be possible to allow students to complete it by themselves.

3.1 It is risky to walk around with large amounts of cash.
It is also a hassle to have to have the correct amount of money each day — the banking takes too much time.

3.2 1 30 kg
3.2.2 50 g
3.2.3 No – only amounts of 50 grams.
3.2.4 No – only amounts of 50 grams.
3.2.5 The scale needs to show a maximum mass of more than 30 kg, so it can show a maximum or 40 kg or 50 kg.
3.2.6 Of the scales shown, the electronic scale is the only one that has the full range of masses needed.

3.3

Category	Number of items	Costs	Total costs per category
Small letters	**712**	R1,70	**R1 210,40**
Medium letters	**58**	R3,45	**R200,10**
Large letters	**27**	R4,35	**R117,45**
Cylinders (short)	**0**	R4,35	
Cylinders (long)	**137**	R7,55	**R1 034,35**
Parcels	**205**	R24,35 for the 1st kg and R3,15 for every extra kg or part thereof	**R4 991,75**
Econoparcel Fixed specification of size: 458 mm × 324 mm × 100 mm 1 kg or less	**0**	R12,95	
Total cost for today's post	**R7 554,05**		

3.4 Students may wish to include other columns to show other calculations. They will have to calculate R1,70 times various amounts and R1,61½ times various amounts. If your college has access to computers with a spreadsheet package, this is the easiest way to do repeated calculations and is a useful skill for students to have. You can also use this activity to let students practise using the memory function on electronic calculators. If your college does not have these facilities, then let students make tables in which they first record multiples of these two costs.

Category of post	Normal cost per individual item.	No. of R1,70 stamps needed	Other stamps needed	Cost of standard mail stamps needed	Total postage cost	Savings per item
Medium letter	R3,45	2	2 × R1,70 = R3,40. R3,45 – R3,40 = 5c	2 × R 1,61 ½ = R 3,23	R3,23 + 5c = R3,28	R3,45 – R3,28 = 17c
Large letter	R4,35	2	2 × R1,70 = R3,40. R4,35 – R3,40 = 95c 1 × 50c 2 × 20c 1 × 5c	2 × R1,61½ = R 3,23	R3,23 + 95c = R4,13	R4,35 – R4,13 = 22c
Short cylinder (students should see that this is the same calculation as above)	R4,35	2	1 × 50c 2 × 20c 1 × 5c	R 3,23	R4,13	22c
Long cylinder	R7,55	4	4 × R1,70 = R6,80. R7,55 – R6,80 = 75c 1 × 5c and 7 × 10 c	4 × R1,61½ = R6,46	R6,46 + 75c = R7,21	R7,55 – R7,21 = 34c
parcel 1kg	R24,35	14	14 × R1,70 = R23,80. R24,35 – R23,80 = 55c 1 × 50 c and 1 × 5c	14 × R1,61½ = R22,61	R22,61 + 55c = R23,16	R24,35 – R23,16 = R1,74
Parcel 2 kg	R24,35 + R3,15 = R27,50	16	16 × R1,70 = R27,20. R27,50 – R27,20 = 30c 3 × 10 c	16 × R1,61½ = R25,84	R25,84 + 50 c = R26,14	R27,50 - R26,14 = R1,36
Parcel 5 kg	R24,35 + 4 (R3,15) = R37,10	21	0	21 × R1,61½ = R33,92	R33,92	R37,10- R33,92 = R3,18
Parcel 10 kg	R24,35 + 9 (R3,15) = R52,85	31	0	31 × R1,615 ½ = R50,07	R50,07	R52,85 – R50,07 = R2, 63

Another calculation that you could ask students to do is to find out the savings they would get if they used a franking machine. You get 4 % discount on franked letters—but there is also the cost of hiring the machine.

So it is probably cheaper to buy standard mail stamps in bulk.

3.5 This template should have the following:

- Letters drawn with exactly the following dimensions:
- 120 mm × 235mm – inside the letter show the dimensions and label it "small letter".
- 176mm × 250 mm – inside the letter show the dimensions and label it "medium letter".
- 250 mm × 353 mm – inside the letter show the dimensions and label it "large letter".

It is best if this is done in nested letters so that it is easy to see which size the letter fits into by placing it once only.

- Attach two ribbons showing 520 mm and 620 mm – this saves making the board long enough to draw this length.
- Circle with diameter 70 mm.
- Slots cut out of the following dimensions:
- 5 mm width and at least 60 mm height – this means that the middle of the small letter's width can be measured.
- 10 mm width and at least 88 mm height – this means that the middle of the medium letter's width can be measured.
- 30 mm width and at least 125 mm height – this means that the middle of the large letter's width can be measured.

Illustration???

Activity 3 — Working in a mail room

You have been assigned to help out in the mail room of a company that handles a lot of letters and small parcels. In this activity we look at some of the services offered by the South African Post Office and also the related costs of the services. This information is given in the Handout 3.

Many people use the Post Office and you can sometimes stand in long queues. It saves time (which means your trouble and the company's money) to have your mail presorted before you arrive at the counter. It is much more comfortable for you to sort the mail at the office (where you can choose your radio station) than to stand in the queue (where you have to watch the Post Office commercial television).

In this activity we will use information provided by the South African Post Office in their brochure: *Rates 2004/5* to "sort and prepare" post to leave the mail room.

At the Post Office counter one pays cash for stamps and postage. If you simply take all the letters and parcels to the counter each day, you will need to carry large amounts of money on you.

3.1 What problems can occur if you have to buy stamps for a large number of letters every day?

3.2 Look at the scales (or mass meters) in the Handout 3. There are electronic scales and "manual" scales. The manual scales have different gradations. Different scales are useful for measuring different masses: for example, you would not use a bathroom scale to measure the mass of a bull.

Now look at the masses mentioned in the postal rates table.

3.2.1 What are the largest masses you need to weigh?

3.2.2 What are the smallest masses you need to weigh?

3.2.3 Do you need a scale that shows parts of a gram?

3.2.4 Do you need a scale that shows grams?

3.2.5 Do you need a scale that can show a maximum mass of more than 1 kg? 2 kg? 5 kg? 10 kg? 20 kg? 30 kg? 40 kg? 50 kg?

3.2.6 Now look at the scales or mass meters. If you could only choose one mass meter for the mailroom, which one would you choose.

3.3 Look at the mass meters on the back of Handout 3.

3.3.1 Write down the masses, to the nearest gram,of the letters, cylinders and parcels shown.

3.3.2 At what size does a letter become a small parcel?

3.3.3 Fill in the table below to show how many of each category of mail you have today.

Category	Number of items	Costs	Total costs per category
Small letters		1,70	
Medium letter		3,45	
Large letters		4,35	
Cylinders (short)		4,35	
Cylinders (long)		7,55	
Parcels		24,35 for the 1st kg and 3,15 for every extra kg or part thereof	
Econoparcel Fixed specification of size: 458 mm × 324 mm × 100 mm		12,95	
Total cost for today's post			

3.3.4 Calculate how much you will pay for each category of postage today.

3.3.5 Calculate the total mail cost for the day. Fill it in on the table.

3.4 You can use standard mail stamps for small letters. You can also use standard mail stamps on other postage, but you may need to add other stamps to make up the correct amount of postage. One standard mail stamp costs R1,70 and it can be used for R1,70 worth of mail. The second table on Handout 3, shows that you get discounts on standard mail stamps if you buy them in large amounts.

3.4.1 Calculate how many standard mail stamps you need for each of the following items. Then work out how much money you would save if you bought standard mail stamps in box of 1 000 at a time. Fill in your answers on the table on the next page.

Category of post	Normal cost per individual item.	Number of R1,70 stamps needed	Other	Cost of	Total postage cost	Savings
Medium letter	R3,45					
Large letter	R4,35					
Short cylinder	R4,35					
Long cylinder	R7,55					
parcel 1kg	R24,35 for the 1st kg and 3,15 for every extra kg or part thereof					
Parcel 2 kg	R24,35 + R3,15					
Parcel 5 kg	R24,35 + 4 (R3,15)					
Parcel 10 kg	R24,35 + 9 (R3,15)					

3.5 If you had to measure the length, breadth and thickness of each letter each day it would be very boring and take a lot of time. It is much easier to have a template of these sizes drawn to 100% scale, so that you can place any letter you are not sure of against the template.

3.5.1 Have you seen such a template at the post office?

3.5.1 Design your own template to measure letter sizes. Make the template on a piece of cardboard.

Letter and parcel postage rates for South Africa

Category	Minimum Size (mm)	Maximum Size (mm)	Maximum Thickness (mm)	Maximum Mass	Cost (Rands)
Small letters	90 × 140	120 × 235	5	50 g	1,70
Medium letters		176 × 250	10	250 g	3,45
Large letters		250 × 353	30	300 g	4,35
Cylinders (short)		520	70	1 kg	4,35
Cylinders (long)		620	70	1 kg	7,55
Parcels		Maximum length 1m Maximum length + girth 2m		30 kg	24,35 for the 1st kg and 3,15 for every extra kg or part thereof
Econoparcel Fixed specification of size: 458 mm × 324 mm × 100 mm	458m × 324 × 100	458 × 324	100	1 kg	12,95

From: *Rates 2004/5 produced by SA Post Office Marketing*

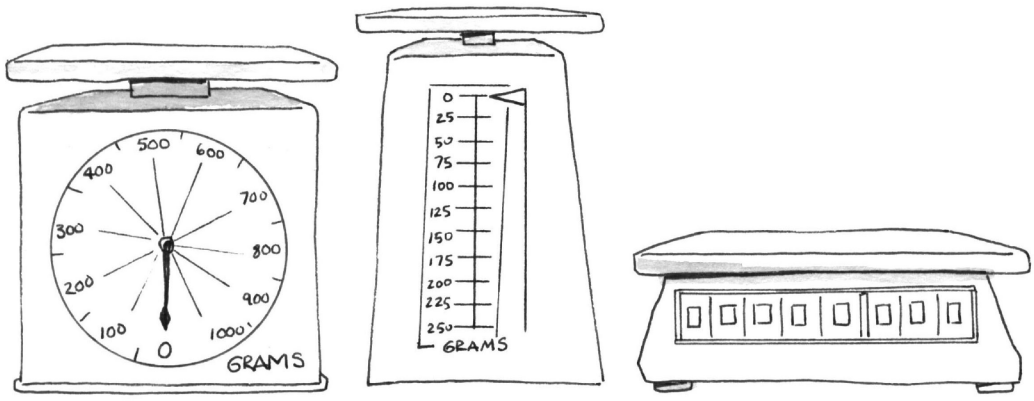
Standard Mail Stamps

Standard mail stamps cost R1,70. A booklet of 10 stamps costs R17,00. You get discounts if you buy standard mail stamps in bulk.

Number of standard mail stamps	Discount	Net Price
Rolls or boxes of 500 stamps	2 %	R 833,00
Rolls or boxes of 1 000 stamps	5 %	R1 6 15,00

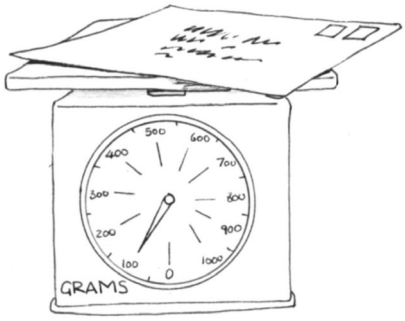
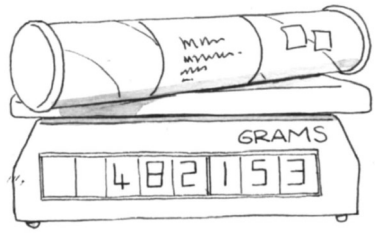
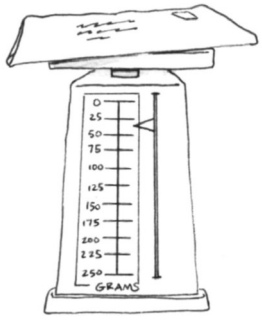
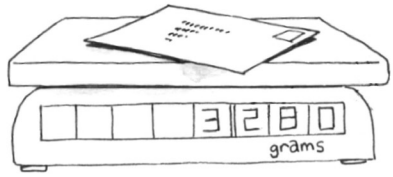
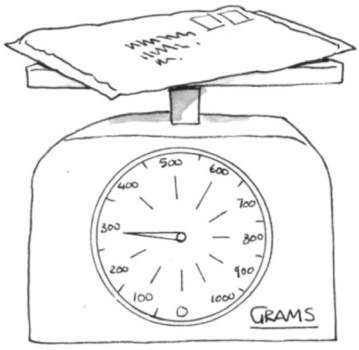
From: *Rates 2004/5 produced by SA Post Office Marketing.*

Types of scales or mass metres



A B C

Today's mail: sizes, masses and numbers

Size of items: 225mm × 320 mm × 11 mm Mass: shown below:  Number of items: 27	Size of items: 550mm × 70 mm Mass: shown below:  Number: 137
Size of items: 110mm × 220 mm x42 mm Mass: shown below:  Number: 439	Size of items: 90mm × 152 mm × 2 mm Mass: shown below:  Number of items: 273
Size of items: 170mm × 250 mm × 8 mm Mass: shown below:  Number: 58	Size of items: 250mm × 320 mm × 40 mm Mass: shown below: Number: 205

Activity 4 — Reading a plan of an office

ABOUT THIS ACTIVITY

Many business administrators are responsible for the purchasing and placement of office furniture. This involves knowing what sorts of furniture are available and how to best place these in an office. In this activity students are exposed to the core measurement skills that office administrators need to be able to do this.

In this activity students not only work with the numbers and space but they also need to make sense of them in terms of the context (see for example question 4.4).

This activity prepares students to meet the following outcomes of Unit Standard 12444:

- SO1: Estimate, measure, and calculate physical quantities to solve problems in practical situations: AC2, 3,6,7

This activity also prepares students to meet the following outcomes of Unit Standard 8982:

- SO1: Demonstrate understanding of and carry out calculations with rational numbers and irrational numbers: AC1, 2, 3, 4, 5, 6
- SO2: Demonstrate knowledge and understanding of the different representations of rational numbers : AC2, 4
- SO3: Demonstrate understanding of and use scientific notation: AC3

MANAGING THIS ACTIVITY

Students should be given Worksheet 4 and a copy of Handout 4. The first page of the worksheet provides a simple plan of an office. There are three skills students need to be able to understand the plan:

- o They need to be able to relate objects in the plan to the key given below the plan and to know some of the standard conventions on a plan e.g. how windows and doors are represented.
- o They need to be able to work with the scale so that they have some sense of the actual size of the real objects.
- o They need to understand that a plan is a view from above and to be able to visualise what the actual objects look like.

If students are unable to visualise the objects in the plan, then do a range of activities in which you first ask them to match pictures drawn from an oblique and side view, then later also a top view (the top view is the most stylised and symbolic). Once they are comfortable with matching views from different perspectives, ask them to draw objects from different views, including top views. Finally, come back to this plan, other plans and simple maps.

A very useful skill is to be able to estimate the size of objects and space around one so that one has a sense of how best to place an objects in that space. Talk with students again about how to use body measures and other everyday referents to estimate size. Let them have some practice estimating the size of your classroom and various objects in it.

4.1 Student should point these out or label them on the drawing.

4.2.1 1:100

4.2.2 The easiest way for students to calculate the dimensions of the real objects is for them to realize that 20 mm on the plan = 1m in reality, so every 20 mm on the plan represents 1 metre. So the can measure the objects in millimetres, halve the number and give the answer in metres.

- Desks: 1,5 metres long x 0,75 metres wide
- Chairs 0, 7 metres across length of seat x 0,5 metres across width of seat
- Photocopier: 0,6 metres long and 0,6metres wide
- Doors width 1m wide
- Window: 1 metre wide
- Bookshelves: 1,2 metres long and 0,3 metres wide/deep
- Stationary cupboard: 1,2 metres long and 0,6 metres wide
- Filing cabinets: 0,5 metres long and 0,7 metres wide

4.3 An example of how students can fit in the furniture is provided at the end of this discussion.

Students need to think about not only the physical size of the furniture but also the human activity that takes place in, on and around the furniture.

Once students have worked out from the plan how they can place the furniture talk to them about whether this makes sense in terms of a real office. Let them consider some of the constraints of an office.

- Computers need to be near plugs – it is not safe to have a spaghetti network of leads running all over the office floor.
- Many people like to sit near a window to get fresh air and light.
- Some people do not like to look directly into the faces of colleagues all the time.
- Other people type confidential material and so their computer screens should not be in full view of the rest of the office.
- Photocopiers do emit fumes which can be unhealthy if breathed in all day long.

Let students check each other's plans to see if they need to adjust any of the placements of furniture.

4.4.1 Length × depth × height measured in millimeters. It means that the length of the piece of furniture is stated first followed by the dimensions of the width/breadth/depth followed by the height. All measurements are given in millimeters.

Discuss with students that an office furniture catalogue has dimensions in length, depth and height, whereas in mathematics we often talk about length, width/breadth and height. If you talk about the width of a filing cabinet or bookshelves it may not be clear which measurement you are referring to. However, it is clear which measurement is the depth and which is the length.

4.4.2 The plan does not show the heights of the pieces of furniture.

4.5 The depth or width and the height of the desks are all the same: only the length is different.

Check that students understand what the word "dimensions" means.

In the next few questions students will need to round off the measurements given in the catalogue table to the approximate lengths they got from the scale diagram.

4.6 Desk b – students need to realize that the scale drawing is a reduced representation of reality so the exact numbers of millimetres cannot be shown. They must round off the measurements from the catalogue.

4.7

- Desk a is just over 1 metre long
- Desk b is about 1,5 metres long
- Desk c and desk b are between 1,5 metres and 2 metre long.
- Students need to pace out the measurement practically – but it is important that students are able to do this as this is one of the best estimates of furniture size and the starting point to seeing if it will fit in the given space. Also remind them of the objects and body measures that they can use as referents and help them to develop others.

4.8 The circular table has a diameter of 1,2 metres and a height of 0,725 metres. Students should note that almost all table, desk and pedestal furniture has the same height.

4.9 Bookshelves b and d

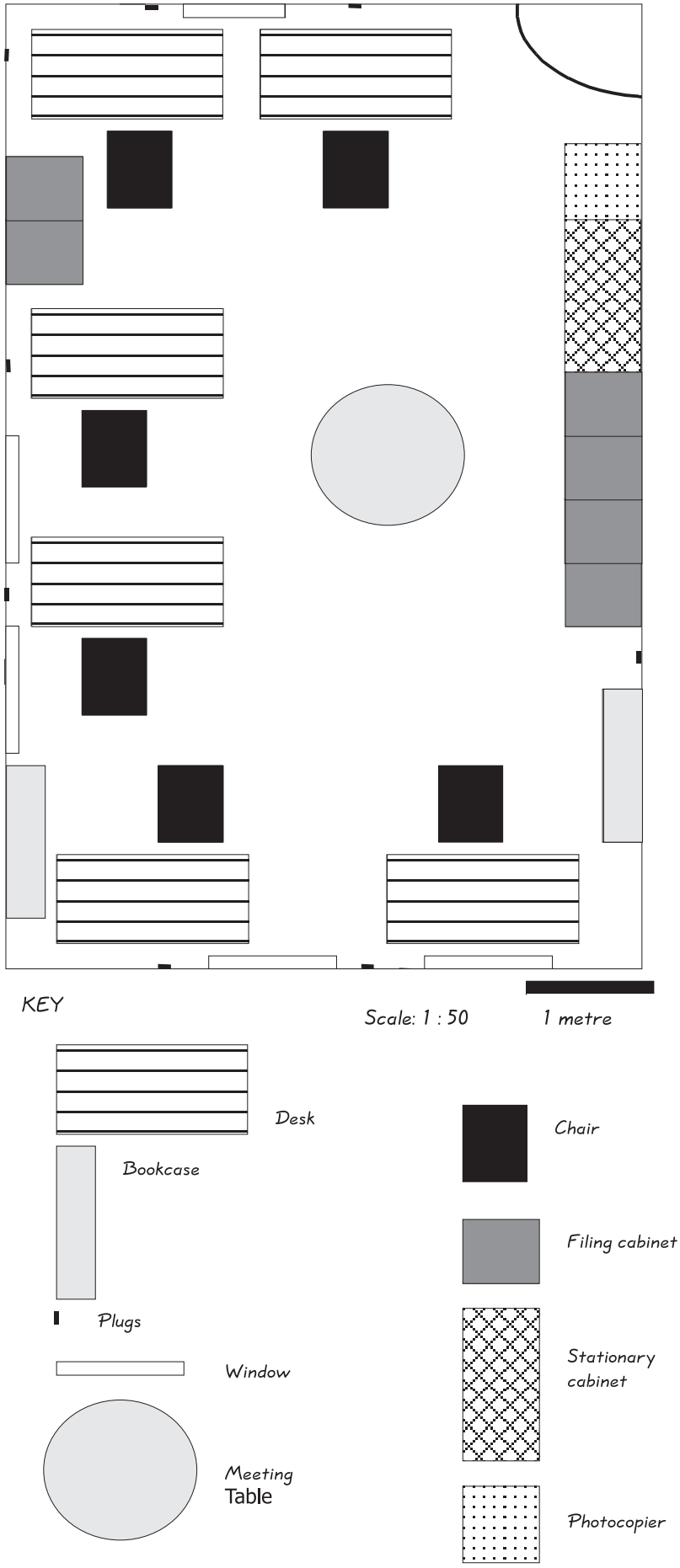
4.10 Bookshelves c and d

4.11 Bookshelf a has narrower shelves and bookshelf b has deeper shelves. If you need to put files (lever arch type files) or wide books on the shelf then choose bookshelf b otherwise choose bookshelf a.

4.12 Windows are often set at about 1 metre above the ground or slightly less

4.13 Bookshelves a & b will fit below most windows, bookshelves c & d will fit below many windows.

See one possible answer to Question 4.3 on the following page.


KEY
Scale: 1 : 50
1 metre
Desk
Chair
Bookcase
Filing cabinet
Plugs
Stationary cabinet
Window
Meeting Table
Photocopier

Activity 4 — Reading a plan of an office

Three temporary staff are joining your administration team to help with your year end busy period. They will need desks, chairs, waste paper baskets, telephones and computers. All of this equipment can come out of the office storage. Your task is to find a way of fitting their furniture into your open plan office space. Remember all of you will still need to be comfortable, if people's working conditions are too cramped it can affect their pace of work.

You will probably need to move quite a lot of heavy office furniture around so it is best to get help from the people who work in the storage section. You also need to check that the new office arrangement does not upset any of your present colleagues. This means that you need to plan how to fit everyone and their furniture in, before the new furniture and office movers arrive. In the handout we give a plan of the office space. This plan includes the present furniture.

4.1 Find the following things in the office plan on Handout 4:
- Desks
- Chairs
- Photocopier
- Doors
- Windows
- Book shelves
- Stationary cupboards
- Filing cabinets
- Electrical plug points

4.2 Below the sketch a scale is given.

4.2.1 Write down the scale in its ratio form.

4.2.2 Use the scale to calculate the real size (length and breadth) of the following objects:
- Desks
- Chairs
- Photocopier
- Doors
- Windows
- Bookshelves
- Stationery cupboards
- Filing cabinets

4.3 You need to fit in another 3 desks of the same size as the others. One way to do this is to make small cut-outs of the furniture and place it in new places on the diagram. This allows you to move things around until you get the most pleasing fit. Do this and then draw the new office plan.

Now that you have a plan of where to put the big pieces of furniture, let's think about how some aspects of human nature limit where you can put furniture in an office. Let's also think about how whether there is anything about certain pieces of furniture that limit where you can place them. Can you think of any things that limit where you can put the furniture? Here are a few:
- o The computers need to be near an electrical wall plug. The length of the lead attached to the computers is about 1 metre.
- o It is not healthy to sit next to a photocopier all day long.
- o Most people like a bit of privacy: they don't like to look directly at someone else all day long.

Do you need to adjust where you placed the furniture?

4.4 A few months later you are asked to order new office furniture. The desks and cupboards should all be the same size. Table 2 of the handout has sizes of furniture from an office catalogue book.

4.4.1 Explain what D × L × H mm means.

4.4.2 Which of these three measurements or dimensions is not shown on the office plan?

4.5 All three desks from the catalogue have two of the dimensions the same. What dimension is different on the three desks?

4.6 Which desk from the catalogue has the same size as the ones shown in the office plan?

4.7 Which desk from the catalogue is:
- just over 1 metre long?
- about 1,5 metres long?
- between 1,5 and 2m long?
- pace out each of the lengths of the desks.

4.8 The measurements of the circular meeting table are given as 1200dia x 725. What does this mean?

With the rest of the class make a circle to show how big the circular table top is.

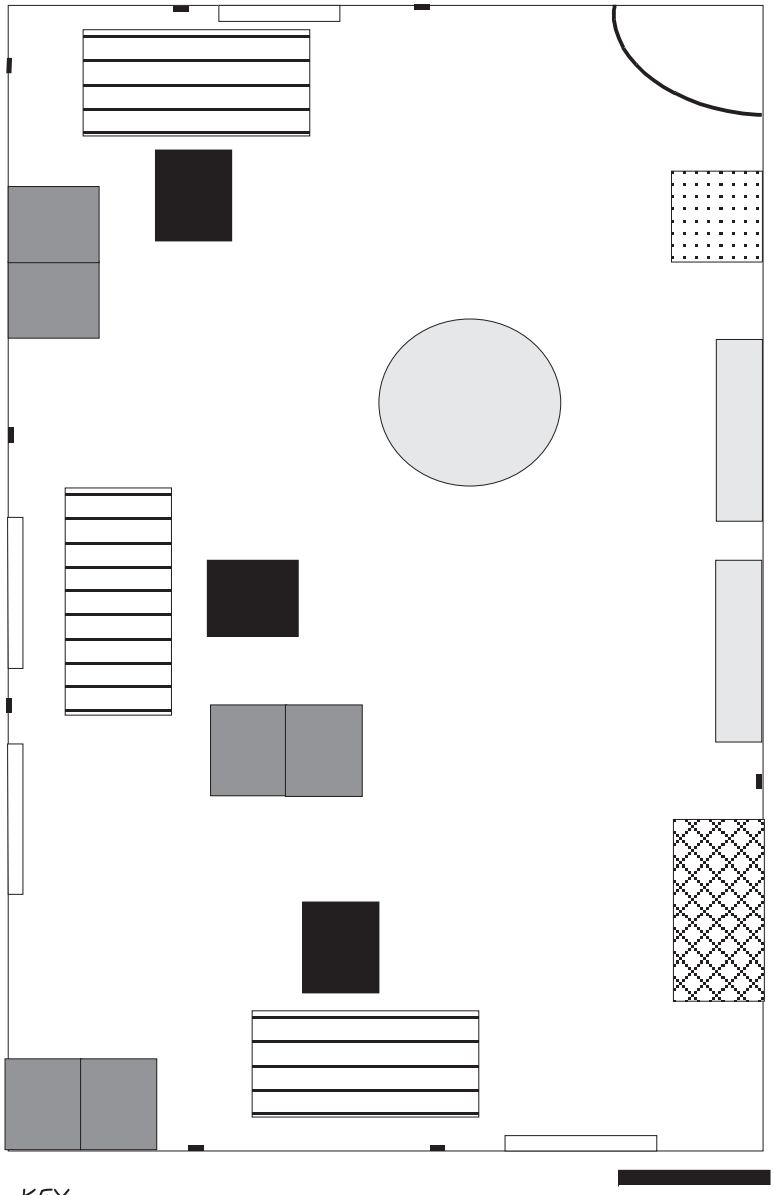
4.9 Which bookshelves from the catalogue are the same size as the ones in the plan?

4.10 Which two bookshelves from the catalogue list are about 1m high?

4.11 The two bookshelves above have one dimension different. What would make you choose one of these bookshelves and not the other?

4.12 In many buildings windows are about 1m high and doors about 2m high. Many windows are set at a standard height from the floor: is this less than 1m, about 1m or more than 1m?

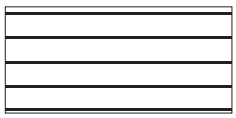
4.13 Which two bookshelves from the catalogue list fit below a window?



KEY

Scale: 1 : 50

1 metre

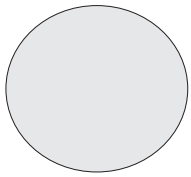


Desk

Bookcase

Plugs

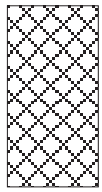
Window



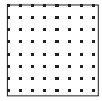
Meeting Table

Chair

Filing cabinet



Stationary cabinet



Photocopier

Dimensions of office furniture

Desks:

	L × D × H mm
a	1228 × 746 × 725
b	1532 × 746 × 725
c	1786 × 746 × 725

Circular meeting table:
1200 dia × 725

Filing cabinets

	L × D × H mm
Four drawer	458 × 622 × 1320
Three drawer	458 × 622 × 1009
Two drawer	458 × 622 × 698

Stationary cupboards:
1320 × 622 × 1640

Book shelves

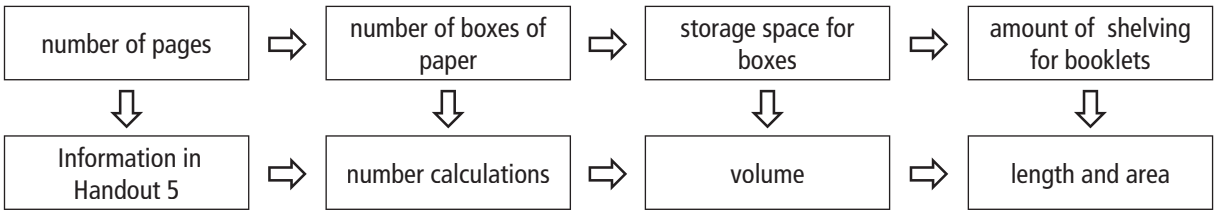
	L × D × H mm
a	915 × 229 × 698
b	1180 × 275 × 698
c	1180 × 229 × 912
d	1180 × 275 × 912
e	1320 × 229 × 1200
f	1320 × 275 × 1840

Activity 5 — Planning for bulk photocopies

ABOUT THIS ACTIVITY

Planning the purchasing and storage of office resources is a key part of office administration. Business administrators need the skill of predicting when and in what quantities things will be needed, in order to purchase stock timeously. They also need to plan for the storage of stock.

In this activity students are asked to convert estimated pages of photocopies into numbers of boxes of photocopy paper. They are then asked to think about the amount of space they will need to store these boxes of paper. Finally, they are asked to calculate how much shelving they will need for the temporary storage of the photocopied books.



In this activity students will work with basic operations, estimations and approximations, calculating volume and areas, as well as thinking about volume and area in the context of office space. Students also calculate turning arc's using Pythagoras theorem. As is the case in real life, they need to use their common sense about what information to use from the handout provided. They are also asked to see where they can use or adapt a previous calculation instead of repeating the whole calculation.

This activity prepares students to meet the following outcomes of Unit Standard 12444:

- SO1: Estimate, measure, and calculate physical quantities to solve problems in practical situations: AC2, 4, 5

This activity also prepares students to meet the following outcomes of Unit Standard 8982:

- SO1: Demonstrate understanding of and carry out calculations with rational numbers and irrational numbers: AC1, 2, 3, 4, 5, 6
- SO2: Demonstrate knowledge and understanding of the different representations of rational numbers : AC4
- SO3: Demonstrate understanding of and use scientific notation: AC3

MANAGING THIS ACTIVITY

In this activity students will be doing lots of similar and sometimes identical calculations. For this reason it provides an ideal opportunity for students to practice using computer based spread sheet programmes or calculators to assist them with their calculations. Using the available technology is an important business administration skill. You may need to check that students know how to use the memory function on the calculator and the formulae functions on the spread sheet programme.

Before you start with the activity help students to understand the context, as reading through the context and understanding it could prove to be the most difficult part of the task. Once students understand the context give them Worksheet 5 and a copy of Handout 5. Students may need some help understanding how to use Pythagoras theorem in questions 5.20 & 5.21.

5.1 About 468 160 pages will need to be copied for the Numeracy course. Help students to see that the handout provides estimated numbers of pages and not exact numbers of pages.

5.2 Estimation:

1 box has 5 reams. Each ream is 500 pages. A box contains 500 sheets of paper.

468 160 can be rounded up to 500 000.

So the numeracy course will need about 1 000 reams of paper. This is 200 boxes of paper.

Calculation:

468 160 ÷ (5 × 500) = 187,264 boxes.

Students need to round this up to 188 boxes, since you can only buy whole boxes, the original number of pages are estimates and it is better to have some extra paper than to run out of paper.

5.3 About 468 160 pages will need to be copied for the Life Orientation course. Help students to see that this is the same number of pages as the Numeracy course. So they do not need to do any calculations.

Students may ask why the Life Orientation and Numeracy courses have the same number of pages but there are more booklets in the Life Orientation course than in the Numeracy course. You can explain that there are fewer pages in the Life Orientation booklets.

5.4 188 boxes of paper.

5.5 About 398 720 pages will need to be photocopied for the English Additional Language courses.398 720 ÷ (5 × 500) = 159,488 boxes. Students should round this up to 160 boxes.

5.6 About 2 192 960 sheets of paper are needed for the Main Language courses. This is 2 192 960 ÷ 2 500 = 877,184 boxes. Students should round this up to 878 boxes.

5.7 The total number of boxes needed is 878 + 160 + 188 + 188 = 1 414 boxes of paper.

5.8 The size of one box of paper is 310 mm × 250 mm × 225 mm = 17 437 500 cubic millimetres.

5.9 0,0174375 cubic metres. Check that students know how to convert cubic units.

5.10 Volume of 1414 boxes of paper is 24, 66625 cubic metres.

5.11 You will need about 25 cubic metres of storage space.

5.12 3 metres × 3 metres × 1,5 metres = 13,5 cubic metres. Not all the boxes will fit in the spare space. You will need to talk to your manager about finding other storage space of a similar size: 2 × 13,5 = 27 m3.

You may need to check that students know two things:

- The formula for calculating volume.
- How the formula relates to stacking or packing boxes.

The last of these can be practically demonstrated with boxes. You many find that many students only know the formula but are unaware of how it relates to packing layers of rows of boxes.

5.13 Students can first convert the length of the shelf into millimetres. It is 2 000 mm long.

Then they can work out the shelf width needed for each booklet:
width of sheet of paper + 20 mm = 210 mm + 20 mm = 230 mm.

Finally they can work out how many piles of booklets can fit onto a shelf: 2 000 ÷ 230 = 8,6956522. However, you cannot fit parts of a booklet onto a shelf. So the answer is 8 piles of photocopied booklets can fit along a shelf.

5.14 There are 28 different Numeracy booklets. 8 booklets fit on a shelf.

So you need 28 ÷ 8 = 3,5 shelves. But you were asked not to mix Learning Areas on the same shelves. So you need 4 shelves.

5.15 There are 42 different Life Orientation booklets. 8 booklets fit on a shelf.

So you need 42 ÷ 8 = 5,25 shelves. But you were asked not to mix Learning Areas on the same shelves. So you need 6 shelves.

5.16 There are 366 different English Additional booklets. 8 booklets fit on a shelf.

So you need 366 ÷ 8 = 45,75 shelves. But you were asked not to mix Learning Areas on the same shelves. So you need 46 shelves.

5.17 There are 1 716 different Main Language booklets. 8 booklets fit on a shelf.

So you need 1 716 ÷ 8 = 214,5 shelves. But you were asked not to mix Learning Areas on the same shelves. So you need 215 shelves.

5.18 The total number of shelves you need is 215 + 46 + 6 + 4 = 271 shelves.

5.19 There are 5 shelves to a bookcase, so you need 271 ÷ 5 = 54,2 bookcases. But you cannot buy part bookcases, so you need 55 bookcases.

5.20 The amount of space need for the trolley to turn depends on a number of factors. In this example we are fixing the following factors:

- Assume that all the wheels can turn – this is done to simplify the problem and is not necessarily the case for all trolleys and certainly not the case for motor vehicles.
- Assume that the width between shelves is narrow, so that all the turning of the trolley happens at the end of the rows. In reality the wider the space between rows, the more turning can happen between rows and the less turning needs to happen at the end of the row.

You may find it useful for students to model the turning of the trolley visually using diagrams and a rectangular piece of paper.

Find out from students if they remember the theorem of Pythagoras. If necessary explain it. Check that they can read the rectangle as a stylised top view of the trolley. Check also that they can see that the diagonal of the trolley is the hypotenuse of the triangle. Check that they are able to calculate the length of the hypotenuse by taking the square root of both sides of the equation.

5.21 $(\text{hypotenuse})^2 = (50)^2 + (120)^2$.

$(\text{hypotenuse})^2 = 2\,500 + 14\,400 = 14\,900$. Hypotenuse $= \sqrt{16\,900} = 130$ cm.

5.22 The least distance you need on each side of the shelving is 130 cm or 1,3 m.

5.23 You have the following width of room in which to place shelves: 7 m – 2,6 m = 4,4 m. This means that you can put two 2 m long shelves next to each other.

5.24 Students may approach this problem in different ways. Here are some approaches:

Each shelf is 30 cm; allow 70 centimetres between shelves for the trolley (50 cm wide plus 10 cm spare on each side). This means that each shelf needs about 1 m. So you can fit 12 sets of shelves into the room i.e. 12 × 2 = 24 shelves.

5.25 Putting shelves back to back means that you save on space between the rows. Each 4 bookcases would use 130 cm or 1,3 m of the length of the room.

12÷ 1,3 = 9,2. This means that they can fit in a maximum of 9 double rows. 9 double rows contain 36 bookcases.

If student's give this sort of answer, ask them to think about space for the door. This probably means that they will need to remove one set (half a row of shelves).

5.26 There need to find place for 55 shelves. Only 36 or even 34 will fit into one room. They need two rooms that are about 7m × 12 m: although one room can be slightly smaller.

Activity 5 — Planning for bulk photocopies

You are temporarily employed by a publishing company that produces South African school books. Seven out of the 9 provinces in South Africa check all material before it can be bought by schools. Each of these 7 provinces asks to get 5 copies of every book for a panel of people to check the material. This year the provinces have asked to see all Grades R (pre-school/ Grade 0) to Grade 3. All the provinces want the material in the same month. Your company sends photocopies of each of their books.

You have been asked to work out:
- how many boxes of photocopy paper will be needed
- what volume of storage space you need for the photocopy paper
- the area of shelving you need for storing the photocopied books.

Use the information given in Handout 5 to help you answer the following questions.

5.1 How many pages will need to be photocopied for all the Numeracy courses?

5.2 About how many boxes of paper will you need for the copies of the Numeracy material? Estimate first, then calculate.

5.3 How many pages will need to be photocopied for all the Life Orientation courses?

5.4 About how many boxes of paper will you need for the copies of the Life Orientation material? Why do you not need to do any calculations for this question?

5.5 How many pages will need to be photocopied for all the for all the English Additional Language courses. How many boxes of paper is this?

5.6 How many pages will need to be photocopied for all the 1st Language or Main Language courses for each of the 11 official. How many boxes of paper is this?

5.7 How many boxes of paper do you need altogether?

Now let's think about how much space we need for all those boxes.

5.8 What is the size of one box of photocopy paper?

5.9 How much is this in cubic metres?

5.10 How much space will you need to store all the photocopy paper you need for the month?

5.11 Will you be able to fit the boxes in the printing and photocopying room? At present there is 3 metres across by 1,5 metres deep of space. The ceiling is 3 metres high.

Now let's think about storing the photocopied books. Each Learning Area needs to be stored on its own: e.g. do not plan to put Numeracy booklets on the same shelf as Life Orientation booklets. The student books, workbooks, teachers guide and each reader need to be placed in their own pile. When storing the photocopies books, you need to place the flat on the shelves: the photocopies are not sturdy enough to stand upright. Handout 5 gives the size of A4 paper, as well as the size of a single shelf.

5.12 You need to leave a gap of about 20 mm between each photocopied booklet, how many booklets can you fit onto one shelf?

5.13 There are 28 different Numeracy booklets altogether. Each booklet needs its own pile. How much shelving do you need for the Numeracy material?

5.14 There are 42 different Life Orientation booklets altogether. How much shelving do you need for the Life Orientation material?

5.15 The English Additional Language course consists of 336 different booklets altogether. Each booklet needs its own pile. How much shelving do you need for the English Additional Language material?

5.16 The Main Language courses consist of 1716 different booklets altogether. Each booklet needs its own pile. How much shelving do you need for the Main Language material?

5.17 How much shelving space do you need in total?

5.18 If your bookcases are 2 metres long and there are 5 shelves per bookcase, how many bookshelves do you need?

5.19 If you have a room that is 7 m x 12 m. Will you be able to put the bookshelves in the room? Remember you need space to walk between the shelves. You use a trolley to carry the photocopies. It is 50 cm wide 120 cm long. It needs a space to be able to turn at the end of each row.

Let's work out how much space you need at the end of the row to turn the trolley.
All the trolley wheels can turn if there is very little space between the shelves then the distance you need to turn the trolley at the end of the row is length of the diagonal of the trolley. The handout shows a scale diagram of the trolley as seen from above. Can you see that the diagonal of the trolley splits the rectangular shape into two triangles? Each of these triangles has a right angle. The side of a triangle opposite a right angle is called a hypotenuse. To calculate the length of the hypotenuse we use Pythagoras' Theorem. It states that $(\text{hypotenuse})^2 = (\text{length of another side})^2 + (\text{length of other side})^2$.

5.20 Write out Pythagoras' Theorem using the correct numbers for the trolley. Then calculate the length of the hypotenuse (which is also the diagonal of the trolley).

5.21 Now state what distance you need on each side of the shelves to be able to turn the trolley.

5.22 How many shelves can you place across the room?

5.23 Now work out how many rows of shelves you can put into the room.

5.24 If you put two rows of shelves back to back you can save space. Explain why this is so.

5.25 Work out how many double rows of shelves you can put into the room.

5.26 Will all the shelves fit into the room? If not how many rooms of this size do you need for the shelves?

Number of provinces that ask for books to be submitted for evaluation: 7

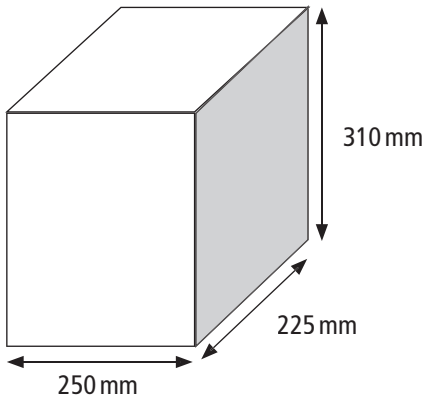
Number of copies per province: 5

Courses

Learning Areas	Number of courses	Translations	Total number of booklets	Estimated total number of pages to be photocopied
Numeracy	4	Yes, one course into 10 languages, one course into 5 languages	28	468 160
Life Orientation	4	Yes, one course into 10 languages, one course into 5 languages	42	468 160
English Additional Language	4	no	336	398 720
Main or 1st Language	2 per language: total 22	no	1 716	2 192 960

Photocopy paper details

One box of photocopy paper has the following dimensions:



A4 paper size = 210 mm × 297 mm

One ream = 500 pages

One box = 5 reams

Dimensions of shelf and A4 paper

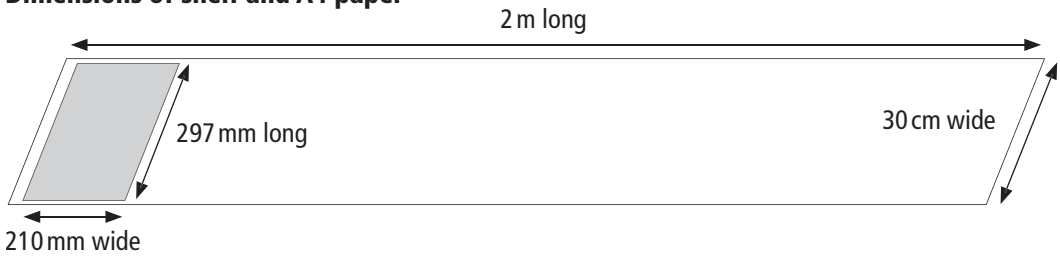
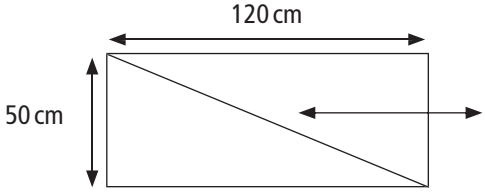


Diagram of trolley



Diagonal of rectangle and hypotenuse of triangle

$(\text{length})^2 = 50^2 + 120^2$

Activity 6 — Making cards using symmetry

ABOUT THIS ACTIVITY

In this activity students use the images of paper cut-out shapes to understand symmetry. The shapes are cut out on the fold lines of "cards", so both the cut-out shape and the "hole" it creates are symmetrical when the card is opened up. Students use these images to understand bilateral symmetry.

Working physically and practically with shapes is the easiest way for students to understand many geometrical concepts. It helps students to create a "bank" of images in their minds on which they can draw to understand concepts. When students have cut a shape on the fold they are asked to imagine what it will look like when they unfold it. This trains them to picture one sort of transformation of shape: the reflection of the shape about the line of symmetry and the combining of the two parts of the symmetrical shape.

Once students have some experience of doing this practically, they can begin to imagine and predict what the "unfolded" shapes will look like when they look at illustrations of shapes cut on a fold.

This activity prepares students to meet the following outcomes of Unit Standard 12444:

- SO1: Estimate, measure, and calculate physical quantities to solve problems in practical situations: AC6
- SO2: Explore transformations of two-dimensional geometric figures: AC1, 2, 5.

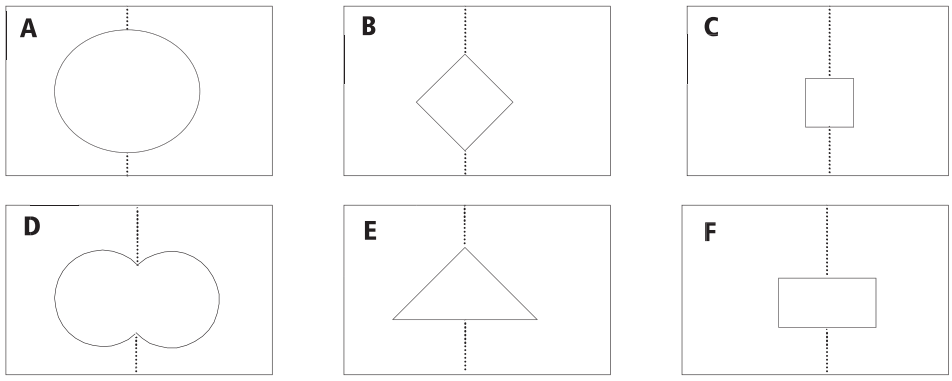
MANAGING THIS ACTIVITY

Students should be given paper and a pair of scissors. Let them first fold the paper and then cut out the shapes on the fold line. Ask them first to draw what the shape will look like when it is unfolded. Then they should unfold the shape and check whether they were correct. Let students each do several examples of cut-out shapes, before starting on the worksheet.

Give students a copy of Worksheet 6 and Handout 6. The images on the worksheet require students to imagine what the cut-out shape will look like. Imagining shapes from pictures is always more technically difficult than working with the physical shape which you can handle, turn over and move. However, it is useful to check that students have developed their prediction skills and that they have understood what they have seen practically.

6.1 Students will make different cut-out patterns — so their answers will be different.

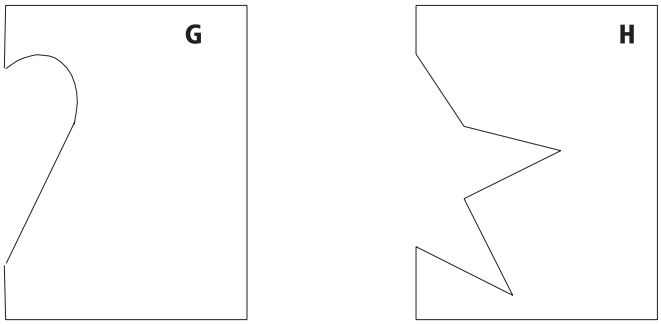
6.2 A – circle B- rhombus C- square



6.3 D- figure 8 on its side E- Triangle F- rectangle

6.4
- A and D both have curved sides.
- A is a full circle and D is two part-circles joined in the middle.
- B and E both have straight sides – some of which are slanting.
- B is a rhombus and E is a triangles. B has 4 sides, E has 3 sides.
- C and F both have 4 straight sides, they are both quadrilaterals.
- C is a square and F is a rectangle.

6.5

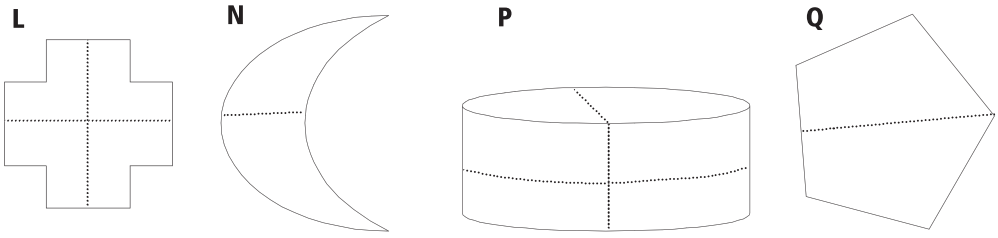


6.6 Check that students understand what symmetry means.

Students' answers will differ here, but some objects that occur often in classrooms and are usually symmetrical are: rulers, windows, blackboards, desks, chairs, blackboard dusters, pencil sharpeners.

6.7 **o Symmetrical shapes:**

L, N, P, Q. Check that students understand that a line of symmetry can be imagined or drawn across the shape in any direction – they should not only think in terms of vertical lines. Some shapes have more than one line of symmetry. Some students may find it difficult to see the line of symmetry in Q – the pentagon. Let them turn the page round, so that the page lies in a landscape direction – then they may see the line of symmetry more easily.



o Asymmetrical shapes: M, O

6.8 Students' drawing will all be different.

6.9 Students' cards will all be different: check to see that they do use symmetry.

Activity 6 — Making cards using symmetry

You are at a week-long workshop at a country venue. You find out that one of the people attending the workshop has a birthday today. Your manager asks you to make a card. There are no shops nearby so you have to rely on the stationery that you have. You do not like to draw, so you decide to make paper cut-out cards.

6.1 Fold a piece of paper and cut out any shape on the fold. Draw what you think the cut-out shape will look like before you unfold it. Now unfold the card to check your prediction.

6.2 Look at the folded cards Handout 6. What shapes will the cut-outs make when you unfold the cards A, B and C?
- o Draw the shape of cut-out A.
- o Draw the shape of cut-out B.
- o Draw the shape of cut-out C.
- o Trace and cut out the shapes to check that your drawings are correct.

6.3 What shapes will the cut-outs make when you unfold these cards D, e and F?
- o Draw the shape of cut-out D.
- o Draw the shape of cut-out E.
- o Draw the shape of cut-out F.
- o Trace and cut-out the shapes to check that your drawings are correct.

6.4 Now let's look at some of their similarities and differences.
- o How is shape A similar fromshape D?
- o How is shape A different from shape D?
- o How is shape B similar from shape E?
- o How is shape B different from shape E?
- o How is shape C similar from shape F?
- o How is shape C different from shape F?

6.5 Fold a piece of paper in half. What shape must you cut on the fold if you want to make a card that will open to show:
- o a heart – see card G on Handout 6.
- o a star – see card H on Handout 6.

In all the shapes above, two sides of the shape are the same. We say that:
- o the sides of the shapes are "mirror images of each other";
- o the shapes have symmetry;
- o the shapes are symmetrical about the line made by the fold; and
- o the line that divides the shape into equal parts is the line of symmetry.

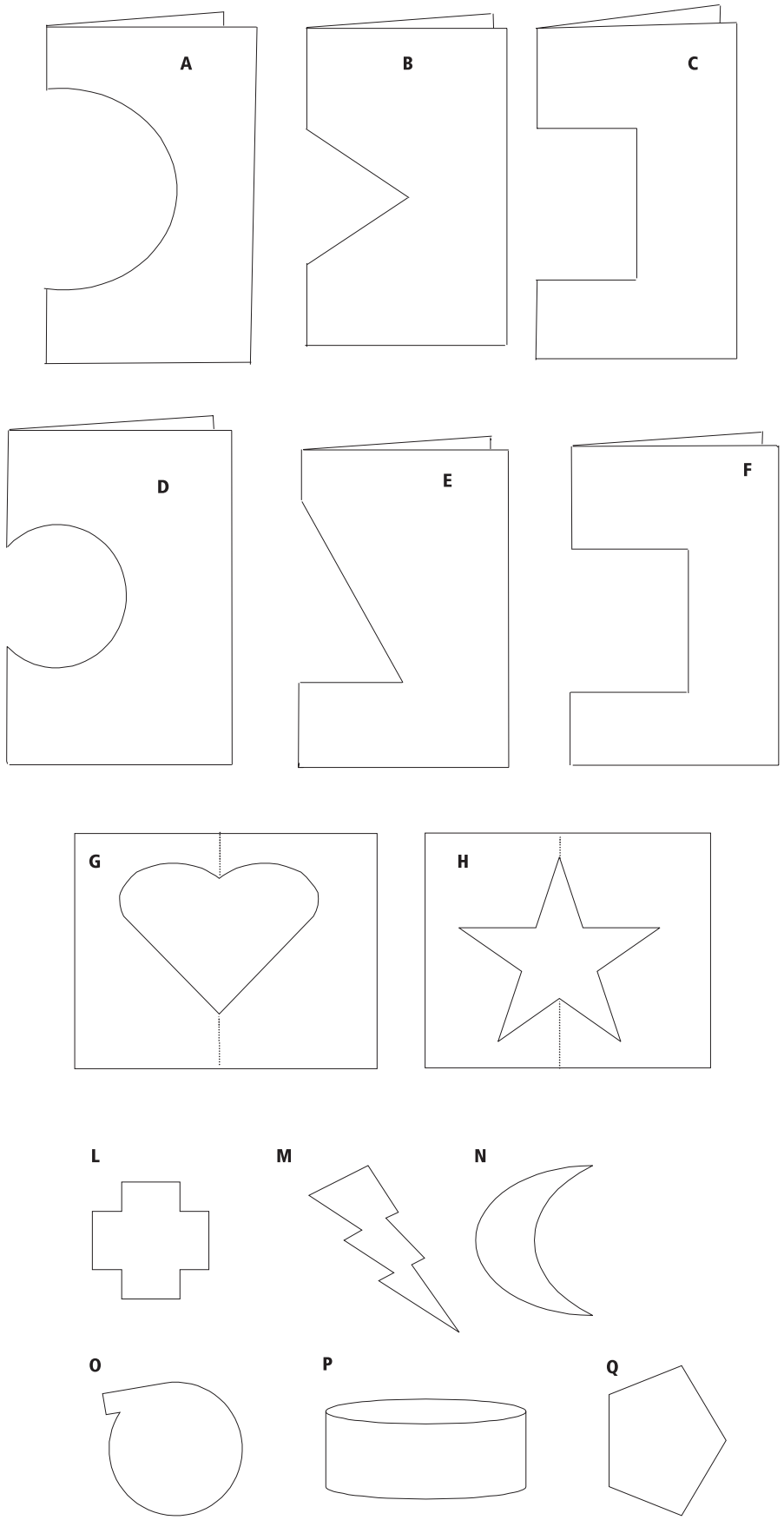
6.6 Look around you find at least 4 objects or shapes that are symmetrical. Draw a picture of each of these shapes and show the line of symmetry.

6.7 Look at shapes L to Q on Handout 6.
- o Which of the shapes are symmetrical?
- o Copy the symmetrical shapes and draw in the line of symmetry.
- o Which of the shapes are not symmetrical?

6.8 When a shape does not have symmetry we say that it is asymmetrical.

Draw 2 more asymmetrical shapes.

6.9 Now make a birthday card using cut-out symmetry shapes.


A
B
C
D
E
F
G
H
L
M
N
O
P
Q

Activity 7 — Making cards by moving shapes

ABOUT THIS ACTIVITY

In this activity students take a simple shape and make patterns by moving the identical shape into different positions. In this way they make up a composite shape or pattern from many repetitions of the identical shape.

In this activity students work with transformations of shapes in which the basic shape stays the same and only its position changes. At level 3 students will apply what they have learned about transformations to the descriptions and analysis of various cultural artifacts. They will also explore the effect of enlargements or reductions on shapes as well as the effect of changing one dimension on measurements of objects and shapes at levels 3 and 4.

By then end of this activity students should be able to:

- Know and use the following mathematical language appropriately – rotations (turns), reflections (flips), translations (slides), templates, superimposition, vertical, horizontal, oblique etc.
- see the repetition of a simple shape in a composite pattern.
- identify how the simple basic shape has been placed and moved to make the pattern.
- make a simple pattern by following instructions about movement of a shape.

This activity prepares students to meet the following outcomes of Unit Standard 12444:

- SO2: Explore transformations of two-dimensional geometric figures: AC3, 4, 5

MANAGING THIS ACTIVITY

These activities are best done practically. Bring cardboard, paper and scissors into class. Student can also explore some of these concepts using potato printing or printing using patterns cut onto erasers. This will mean bringing the potatoes / erasers, paints and brushes to the lesson. Many pressed foam cut-outs are commercially available for craft printing. If you wish students to explore these concepts using printing, then it is important that some place on the outside of the potato or stamp is marked so that students can easily see when the template is turned. Remember that reflections are almost impossible to do when stamping or printing.

Another way in which students can explore these movements is by working with shapes in a computer package. When they drag the shape, it is sliding it. Most computer packages also give the option of flipping and rotating shapes. It is important to realise though that in the activities in this worksheet we rotate the shapes about one of their corners or end points, while computer programmes often rotate the shape about its midpoint. It is important to discuss this difference with students if they are using computer packages.

In general it is probably worth it to use paper/card cut-out templates even if you also use other methods of exploring isometric transformations.

Once students have an understanding of these movements of shapes ask them to look around the classroom and see if they can describe any of the patterns in terms of a movement of some basic shape.

7.1 Students will most easily make sense of this if they trace and cut out the "lightning" shape. They can then move it and place it directly onto each of the positions in each of the patterns. Many students find it difficult to imagine a specific series of movements in their heads, but it is easy to "see" what is happening and how the movements make the pattern if they physically move the shape.

Tell students to imagine that they are printing with the shape. Students may describe the movement in many different ways. First allow them to use their own language before supplying them with the formal language.

7.2 Students may give any of the following answers:

- o Sideways movement.
- o Horizontal movement.
- o Moved across the page.
- o Moved in a straight line.
- o Pulled/pushed the shape.

Once students have had a chance to express the movement in their own words, then give them the two terms "to slide" and "to translate". Mathematically a shape is translated if the lines that join corresponding points are parallel. Students do not need to know this but they do need to be able to recognise when a pattern is made from sliding a shape, and to know that this need not necessarily be only in a horizontal direction but can happen in any direction. It will, however, always be in a straight line.

7.3 Students may describe this as:
- o Twisting it around.
- o Turning it around.

Once students have had a chance to express the movement in their own words, then give them the two terms "turn" and "rotate".

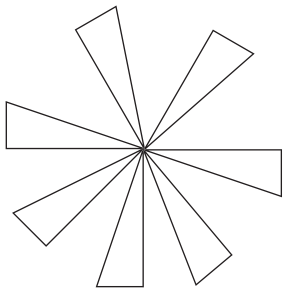
7.4 Student's may describe this as:
- o Turning it over.
- o Flipping it.

Once students have had a chance to express the movement in their own words, then give them the two terms "flip" and "reflect". Ask students to imagine that they have put the edge of their hands on a mirror. Talk about how the reflection is the same and how it is different to their hand. They can also describe or draw reflections in dams or slow moving riversin similar ways. Explain to students that reflections can be in horizontal or vertical planes. Computer programmes are useful to explore what shapes look like when you flip them.

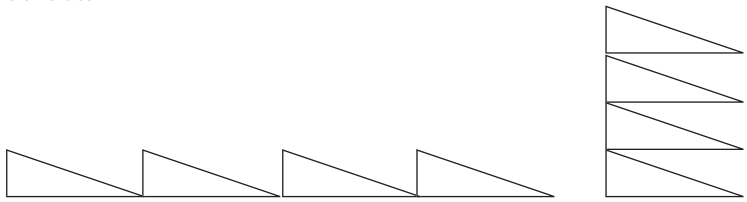
7.5 Here students must use the mathematical language that they have just learned to describe how the patterns are made. First see whether students can imagine the movements in their heads. If they find this too difficult, then they should draw the moon shape onto card and cut it out. They should then move the shape and draw around it in each position to replicate the pattern. Finally, they can describe it.

Talk with students about how some patterns are made by a combination of these movements and not one movement on its own.
- o Pattern 4: rotate (turn).
- o Pattern 5: slide (translate).
- o Pattern 6: slide (translate).
- o Pattern 7: This pattern can be made in different ways:
  - flipping (reflecting only).
  - flipping the basic shape to make the shape, then you can slide (translate) this shape to make a row of them.
- o Pattern 8: this can be achieved simply by reflection (flipping) or it can be achieved by a combination of reflection (flipping) - to make a row or a column - and then translation to make the other rows or columns.
- o Pattern 9: slide (translate).
- o Pattern 10: slide and rotate. Some students may also say that some of the lines of moons can be made from their opposite line by reflection.

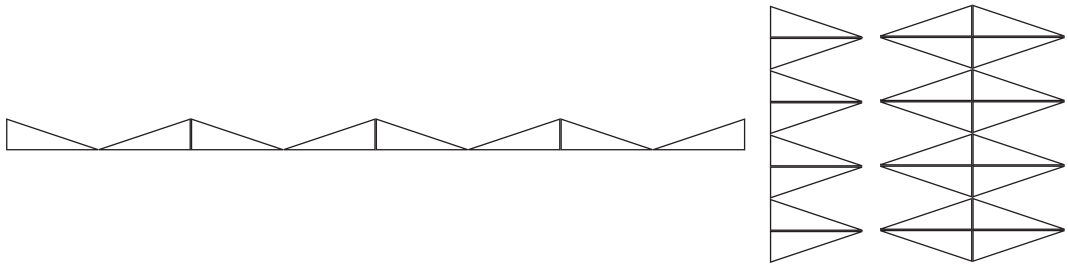
7.6 Students may make correct but different patterns to the sample answers provided below.
- o Rotate:



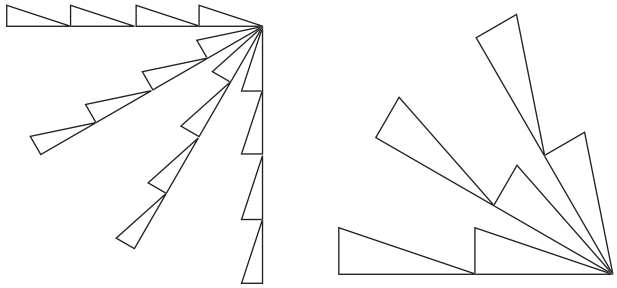
- translate



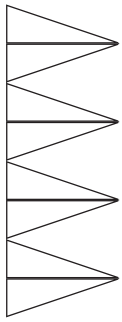
- Reflect



- Rotate

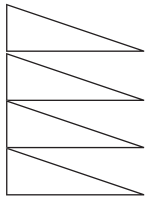


- Reflect vertically



- Reflect horizontally

- Translate vertically



- Slide, slide, slide, reflect

Activity 7 — Making cards by moving shapes

The last set of cards you made were a great success. Everyone's positive comments have motivated you to make some different cards. You still have only your basic stationary to work with. This time, instead of making cut-out shapes, you cut out a basic shape and use this to make repeating patterns in different ways.

7.1 Trace and cut-out the basic shape from page 1 of Handout 7. This shape is called your template.

7.2 Can you see that Pattern 1 is made up of identical shapes?

Place your template over the first shape in pattern 1. Now move the template so that it covers the second and then the third shape.

Explain how you moved the template.

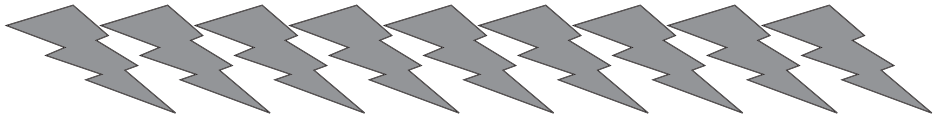
7.3 Imagine that you are able to print with the template. By moving the template in certain ways you can make different patterns. When you put a shape on top of another shape we say that you "superimpose" the one shape on the other.

Superimpose your template over the first shape in Pattern 2. Now move the template to each of the next shapes in turn. Explain how you moved the template to get this pattern.

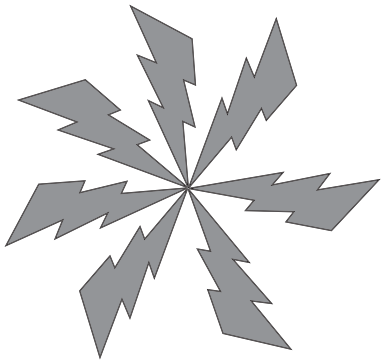
7.4 Superimpose your template on one shape in Pattern 3. Now move the template to each of the next shapes in turn. Explain how you moved the template to make Pattern 3.

In mathematics we talk of different ways of moving shapes. In the Handout 7 the same shape was moved in different ways to create each pattern.

Sliding or translating: To make Pattern 1, you slide the template in a straight line.



Turning or rotating: To make Pattern 2, you need to turn the template around.



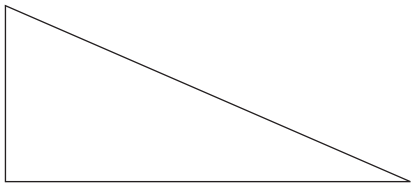
Flipping or reflecting: To make Pattern 3, you need to flip the template over to get from one position to the next.

7.5 Now work with this shape:

Explain how you need to move the shape to make the related patterns on the handout.

- Pattern 4?
- Pattern 5?
- Pattern 6?
- Pattern 7?
- Pattern 8?
- Pattern 9?
- Pattern 10?

7.6 Trace and cut out this triangle:



Make a template by pasting it onto cardboard. Make patterns by moving the shape in the ways stated below. Trace around the outline shape each time.

7.6.1 Rotate / turn.

7.6.2 Translate / slide.

7.6.3 Reflect / flip.

7.6.4 Slide and rotate.

7.6.5 Reflect vertically.

7.6.6 Reflect horizontally.

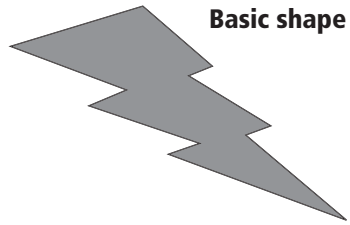
7.6.6.1 Translate vertically.

7.6.7 Slide, slide, slide and then reflect.

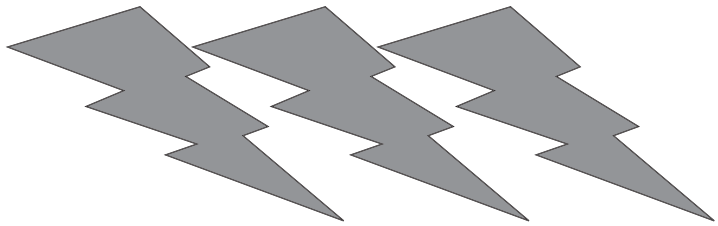
7.7 Now cutout your own template and make 3 patterns with it. Swop your patterns with a partner. Describe to your partner how he/she made his/her patterns.

7.8 Look around the classroom. Describe any patterns that you can see by talking about sliding, flipping or turning.

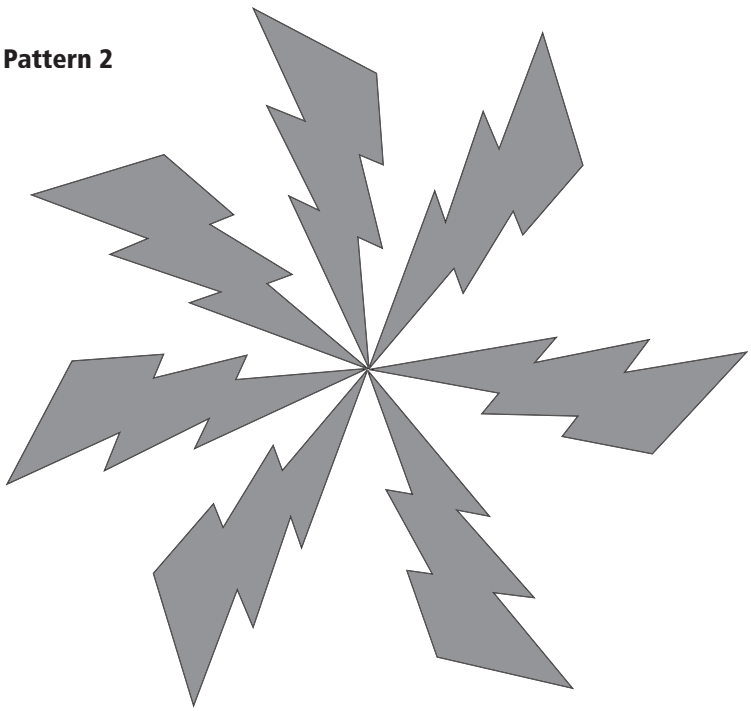
Basic shape



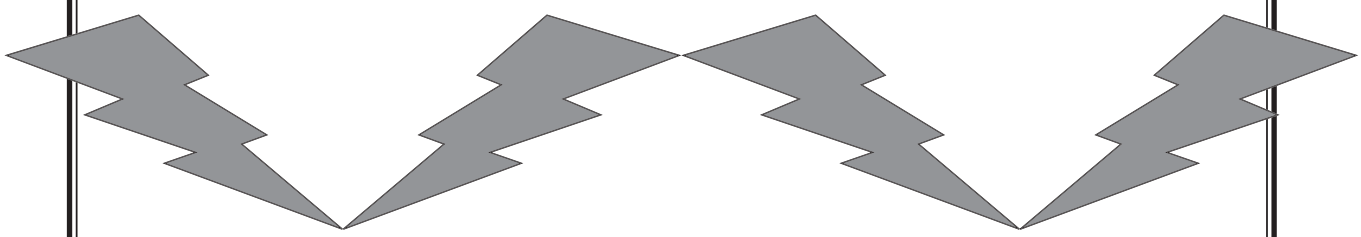
Pattern 1



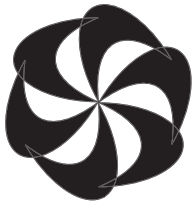
Pattern 2



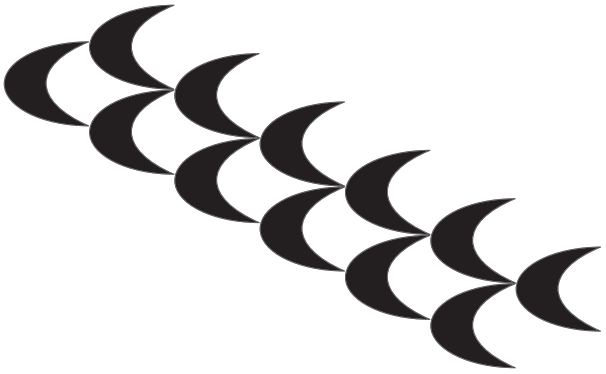
Pattern 3



Pattern 4



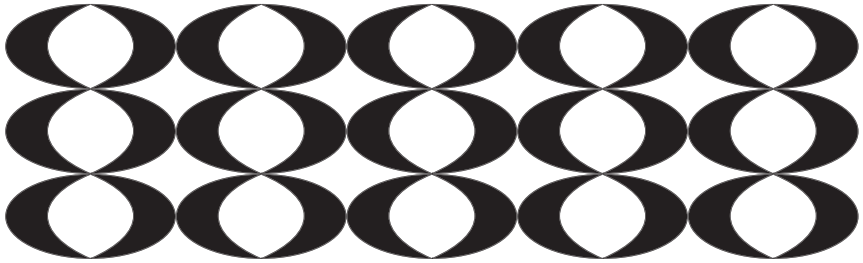
Pattern 5



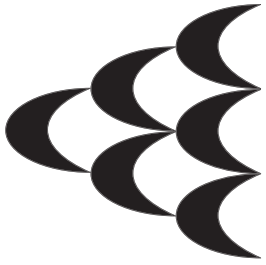
Pattern 6

Pattern 7

Pattern 8



Pattern 9



Pattern 10

Activity 8:
Assessment — Organising drinks for a 21st birthday party

ABOUT THIS ASSESSMENT ACTIVITY

In this assessment students are asked to plan the beverages for a 21st party. Students will be working with capacity. They need to look up information about capacity and costs in Handout 8.

As is the case in many of the calculations in this course students will need to think about how the context impacts on estimations and approximations. They will also need to check whether later calculations impact on calculations and decisions.

Students first need to think about what proportion of people will drink each type of beverage: ratios are provided in Handout 8. Then students calculate how much of each beverage people are likely to drink. They need to convert glasses to millilitres and then factor in the number of people who prefer that beverage. Finally students need to consider what sort of quantities to buy. Different beverages are sold in different quantities. Full bottles, kegs and boxes of juice can be returned. Students need to take into account that it is a 21st birthday party, so the festivities are likely to continue late into the night and people may drink more than normal. They should therefore round up quantities rather than rounding down quantities.

MANAGING THIS ACTIVITY

Students have encountered a similar task in Activity 2. They have also subsequently had a lot of experience working with measurements and shape. Students should be given Worksheet 8 and Handout 8 and asked to complete the task on their own.

8.1
- $\frac{1}{5}$ of 80 people = 16 people expected to drink juice.
- $\frac{3}{5}$ of 80 people = 48 people expected to drink beer.
- $\frac{1}{10}$ of 80 people = 8 people that are expected to drink red wine.
- $\frac{1}{10}$ of 80 people = 8 people that are expected to drink white wine.
- $\frac{1}{5}$ of 80 people = 16 people expected to drink non-alcoholic champagne.
- $\frac{4}{5}$ of 80 people = 64 people expected to drink normal alcoholic champagne.

8.2
- 16 people each drink 4 glasses of 375 ml: 16 × 4 × 375 ml = 24 000 ml. A total of 24 000 ml of juice is expected to be consumed. 24 litres of fruit juice is needed.
- 48 people each drink 5 glasses of 375 ml: 48 × 5 × 375 ml = 90 000 ml. A total of 90 000 ml or 90 litres of beer expected to be consumed.
- Red wine: 8 people each drink 4 glasses of 185 ml: 8 × 4 × 185ml = 5 920 ml. A total of 5 920 ml or 5, 920 litres of red wine is needed.
- White wine: 8 people each drink 4 glasses of 185 ml: 8 × 4 × 185ml = 5 920 ml. A total of 5920 ml or of white wine is needed.
- Non-alcoholic champagne: 16 people each drink 1 glass of 170 ml: 16 × 4 × 170ml = 2 720 ml. A total of 2 720 ml or 2,72 litres of non-alcoholic champagne is needed.
- Champagne: 64 people each drink 1 glass of 170 ml: 64 × 170ml = 10 880 ml. A total of 10 880 ml or 10,88 litres of champagne is needed.

8.3 Part of the key to these answers is that this is a 21st birthday party which is a big celebration and a major party, so students should not undercater and spoil the event. They should round up rather than rounding down (and risk ruining the party).

The decision about the number of barrels needed will be affected by the cost of the different types of barrels. Students should come back to this answer and adapt it once they have completed the next set of questions about costs.

- You need about 24 litres of fruit juice. Juice comes in 5 litre boxes. 5 boxes give 25 litres. 4 boxes will give 20 litres. You don't want to run out, so rather buy 5 boxes.
- 90 litres is needed. You get 30 litre kegs and 50 litre kegs. Three 30 litre barrels will give exactly 90 litres. Two 50 litre barrels will give 100 litres, which allow about 26 more glasses of beer than the calculation allows for. Once students have completed the next set of questions, they will find out that buying two 50 litre kegs is cheaper than buying three 30 litre kegs.
- You need 5 920 millilitres of red wine. This is 7,89333333 bottles of red wine: it makes more sense to buy 8 bottles of red wine.
- You need 5 920 millilitres of white wine. This is 7,89333333 bottles of red wine: it makes more sense to buy 8 bottles of white wine.
- You need 2 720 ml of non-alcoholic champagne. This is 3,6266667 bottles: it makes sense to buy 4 bottles of non-alcoholic champagne.
- You need 10 880 ml of non-alcoholic champagne. This is 14,506667 bottles: it makes sense to buy 15 bottles of champagne.

8.4
- Juice: Five boxes of 5 litres of juice will cost R 174,75.
- Beer: Three 30 litre kegs costs 3 × R 452,54 = R 1 357,62
  Two 50 litre kegs costs 2 × R 620,88 = R1 241,76.
  Because of these costs it makes more sense to order the two 50 litre kegs. Students ought to go back and see whether they need to adjust their answer to the previous question about beer.
- Wine: 8 bottles of red wine @ R42 will cost R 336,00
  8 bottles of red wine @ R30 will cost R 240,00
- Champagne:
  4 bottles of non-alcoholic champagne @ R 23,95 will cost R 95,80.
  15 bottles of champagne at R 29,95 will cost R 449,25.

Total cost: R 174,75 + R 1 241,76 + R 336,00 + R 240,00 + R 95,80 + R 449,25 = R2 537,56.

8.5 Students may mention the need to think about hiring glasses and buying ice.

Activity 8 — Assessment: Organising drinks for a 21st birthday

You are helping to organise your brother's 21st birthday party. You have the task of organising the drinks.

You have invited 100 people. So far 65 people have said that they are coming. You plan for a total of 80 people.

Your task is to work out what different drinks you need to buy and in what quantities you should buy them.

In Handout 8 is a suggested breakdown of what proportion of people you can expect to drink each beverage. Everyone will drink one glass of champagne, but then they will also drink other beverages.

8.1 If you expect 80 people, work out how many people you think will drink the following:
- Juice
- Beer
- Red wine
- White wine
- Non-alcoholic champagne
- Champagne

8.2 Calculate how much of the following beverages you expect people to drink?
- Juice
- Beer
- Red wine
- White wine
- Non-alcoholic champagne
- Champagne

8.3 Calculate the total quantities you should order. Remember that any unopened bottles, boxes or kegs can be returned and your money will be refunded.
- Juice – How many boxes? What flavours?
- Beer – What size keg should you order? How many kegs?
- Red wine – How many bottles?
- White wine – How many bottles?
- Non-alcoholic champagne – How many bottles?
- Champagne – How many bottles?

8.4 Calculate the costs.

8.5 You have now worked out the amounts and costs of the drinks. What else do you need to plan for to complete the task of organising the liquid refreshments for the cocktail party?

Assessment — Organising drinks for a 21st birthday party

Number of people expected: 80

Expected choices of drink

Champagne	Other drinks
$\frac{1}{5}$ non-alcoholic	$\frac{1}{5}$ juice
$\frac{4}{5}$ normal alcohol	$\frac{3}{5}$ beer
	$\frac{2}{10}$ wine; $\frac{1}{2}$ prefer red wine; $\frac{1}{2}$ prefer white wine

Expected quantities to be consumed
Juice – 4 glasses per person: one glass holds 375 ml.
Beer – 5 beers per person: one glass holds 375 ml.
Wine – 4 glasses per person: one wine glass holds 185 ml.
Champagne – 1 glass per person: one champagne glass hold 170 ml.

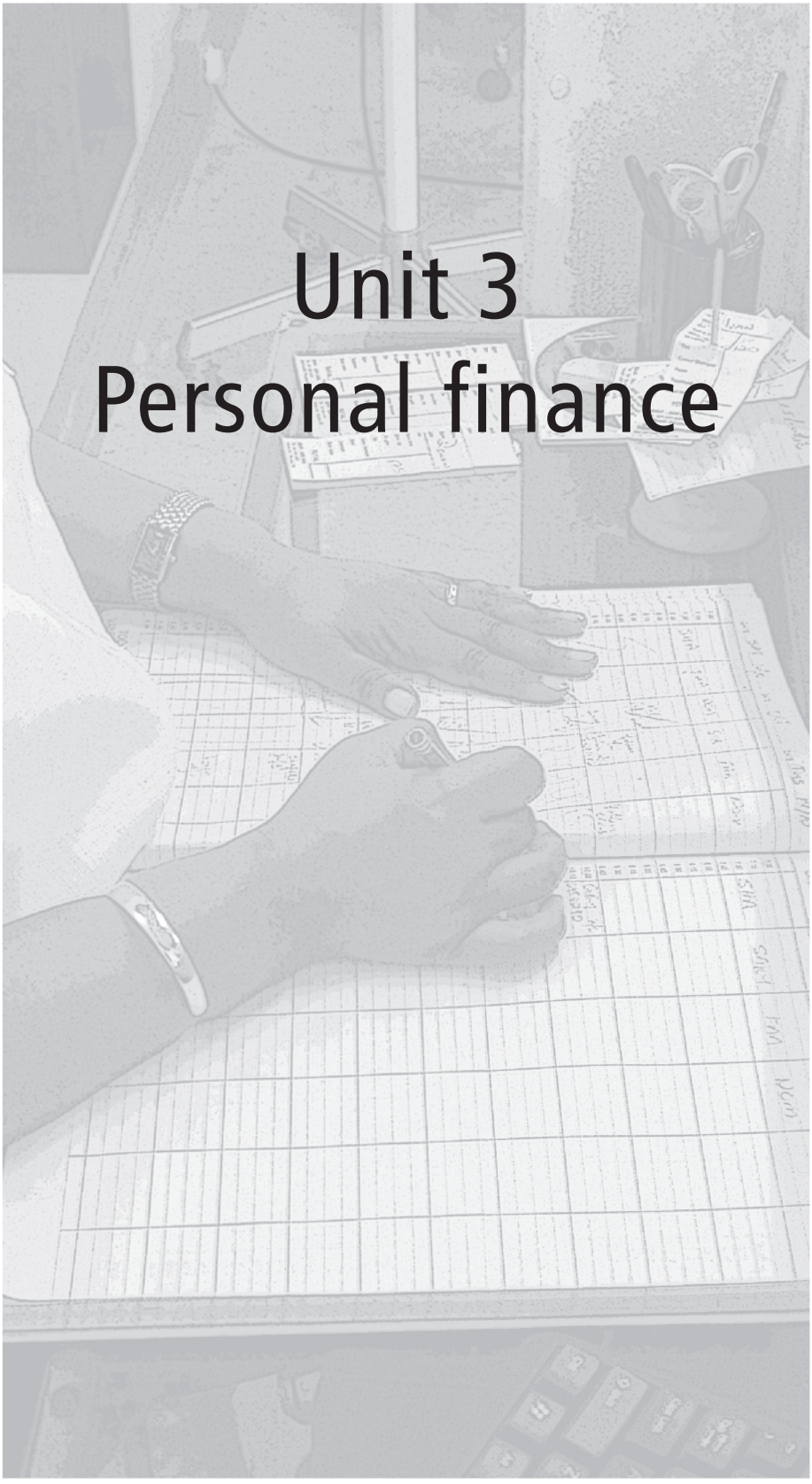
Costs:
Juice @ R34,95 for 5 litres.
White wine @ R30 a bottle. Wine bottles hold 750 ml.
Red wine @ R42 a bottle. Wine bottles hold 750 ml.
Champagne @ R29,95. Bottles hold 750 ml.
Non-Alcoholic champage @ R23,95. Bottles hold 750 ml.

Beer

Size of keg/barrel	Cost	Deposit	Cost including deposit
50 litres	R 620,88	R 330,00	R 950,88
30 litres	R 452,54	R 330,00	R 782,54

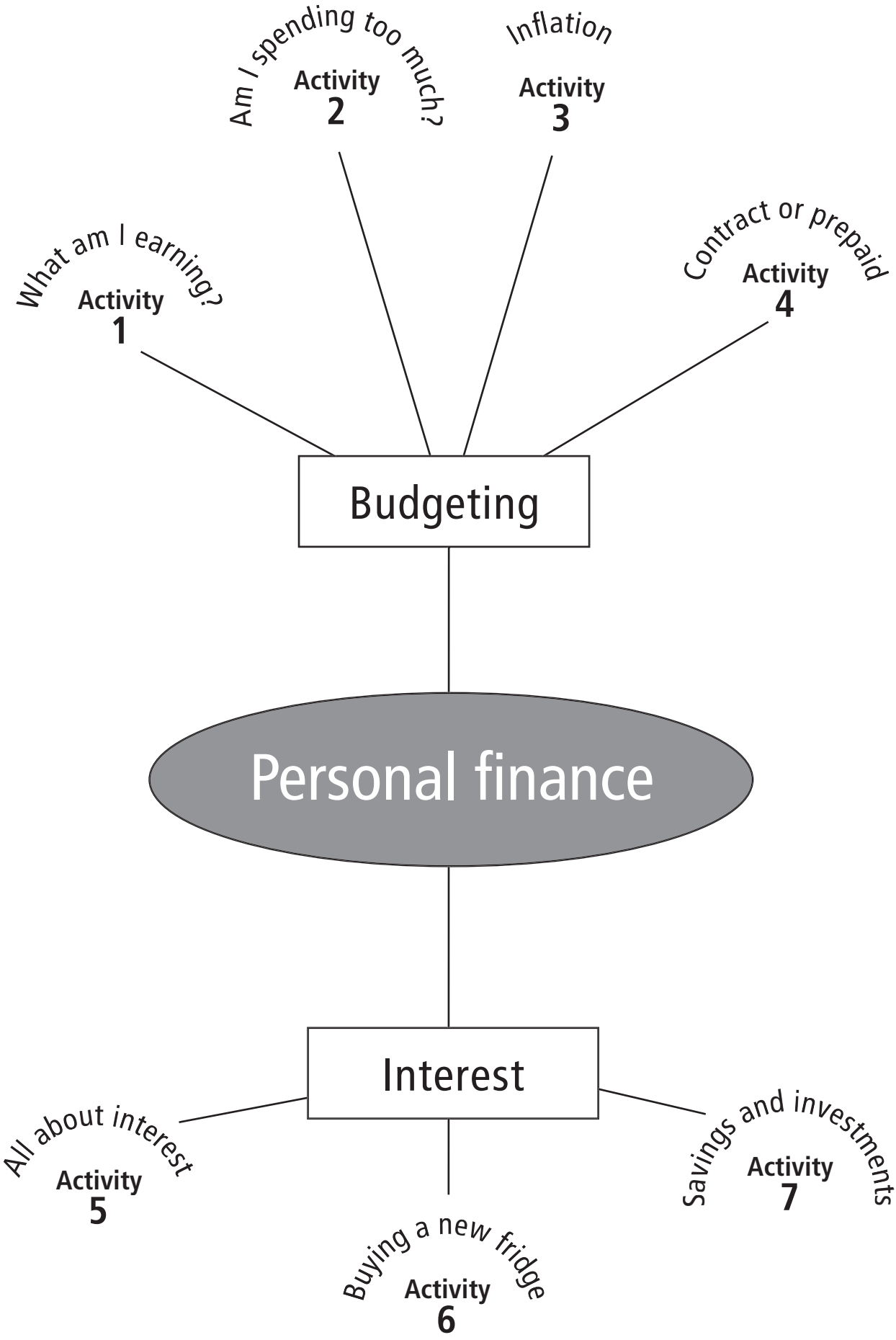
Mathematical Literacy

NQF LEVEL 2



Unit 3
Personal finance

Generic Business Administration


Am I spending too much?
Activity 2
Inflation
Activity 3
What am I earning?
Activity 1
Contract or prepaid
Activity 4
Budgeting
Personal finance
Interest
All about interest
Activity 5
Savings and investments
Activity 7
Buying a new fridge
Activity 6

Overview

This unit uses pay slips, budgeting, cell phone contracts, inflation and buying a fridge to develop mathematical skills including:

- Working with percentages
- Classifying data
- Percentage increase
- Finding averages
- Calculating simple interest
- Calculating compound interest
- Interest rates

In addition to developing the mathematical skills of learners, this unit is also intended to give learners an opportunity to develop an understanding of:

- How to read a pay slip
- How to calculate personal tax
- How to read advertisements
- Inflation
- Budgeting
- Cell phone contracts
- Savings and investments
- Stokvels
- Buying big items on hire purchase

The unit consists of seven activities. The first four deal with budgeting. The next three deal with interest.

Activity 1 — This activity deals with a typical pay slip. We look at the deductions that are taken off monthly and the resulting net income. We also look at the contributions made by a company on behalf of the employee.

Activity 2 — In this activity we look at budgeting. We classify expenses into fixed, changing and irregular expenses.

Activity 3 — This activity deals with inflation. We compare prices of commodities over a ten year period and look at economic indicators.

Activity 4 — This activity deals with cell phone contracts. We look at an advertisement for contracts and prepaid options. We look at the best contract for various situations.

Activity 5 — This activity deals with simple and compound interest. We do calculations to establish what is the best option for a pensioner who has to live off R90 000.

Activity 6 — This activity deals with buying a new fridge. We look at various ways of financing this venture. This activity could be used as an assessment activity.

Activity 7 — This activity deals with savings and investments. We look at stokvels and a variety of bank accounts. We need to be able to choose the best option for our particular situation. We focus on reading interest rate tables supplied by banks.

Unit outcomes

The Following Assessment Standards are addressed by this unit. We know this when the learner:

Uses mathematics to investigate and monitor the financial aspects of personal and community life. (MathLit 7469)

- Uses mathematics to plan and control personal and/or household budgets and income and expenditure. (SO1).
  - Plans describe projected income and expenditure realistically. (AC1)
  - Calculations are carried out using computational tools efficiently and correctly and solutions obtained are verified in terms of the context. (AC2)
  - Budgets are presented in a manner that makes for easy monitoring and control. (AC3)
  - Actual income and expenditure are recorded accurately and in relation to planned income and expenditure. Variances are identified and explained and methods are provided for control. (AC4)
- Uses simple and compound interest to make sense of and define a variety of situations. (SO2)
  - The differences between simple and compound interest are described in terms of their common applications and effects. (AC1)
  - Methods of calculation are appropriate to the problem types. (AC2)
  - Computational tools are used efficiently and correctly and solutions obtained are verified in terms of the context or problem. (AC3)
  - Solutions to calculations are used effectively to define the changes over a period of time. (AC4)

	Use mathematics to investigate and monitor the financial aspects of personal and community life								
		ACTIVITY	1	2	3	4	5	6	7
SO1	Use mathematics to plan and control personal and/or household budgets and income and expenditure.								
	AC1	Plans describe projected income and expenditure realistically.	✓	✓					
	AC2	Calculations are carried out using computational tools efficiently and correctly and solutions obtained are verified in terms of the context.	✓	✓	✓	✓			
	AC3	Budgets are presented in a manner that makes for easy monitoring and control.	✓	✓	✓				
	AC4	Actual income and expenditure are recorded accurately and in relation to planned income and expenditure. Variances are identified and explained and methods are provided for control.		✓	✓	✓			
SO2	Use simple and compound interest to make sense of and define a variety of situations.								
	AC1	The differences between simple and compound interest are described in terms of their common applications and effects.					✓	✓	✓
	AC2	Methods of calculation are appropriate to the problem types.					✓	✓	✓
	AC3	Computational tools are used efficiently and correctly and solutions obtained are verified in terms of the context or problem.					✓	✓	✓
	AC4	Solutions to calculations are used effectively to define the changes over a period of time.					✓	✓	✓

Act 1 What am I earning?
Act 2 Am I spending too much
Act 3 Inflation
Act 4 Contract or prepaid
Act 5 All about interest
Act 6 Buying a new fridge
Act 7 Savings and investments.

Activity 1 — What am I earning?

This activity sets the scene for the unit on personal finance by looking at a typical payslip and highlighting the difference between gross and net income. The next activity will focus on budgeting.

ABOUT THIS ACTIVITY

This activity focuses on vocabulary associated with payslips and works through the types of deductions that come off every month. The aim is for the students to get an understanding of what money they actually have available to spend each month. This is in preparation for the next activity on budgets. They also look at how taxes are calculated. This activity addresses AC1, 2 and 3 of the SO1 of the personal finance unit standard.

MANAGING THIS ACTIVITY

This activity has a worksheet and 2 handouts. Students need to be able to work out percentages and to read tables. Some of the terminology might need to be explained.

1.1 Patricia Prentis

1.2 The Finance Institute

1.3 23 July 2004

1.4 43 months as it is the 43rd payslip. This is 3 years and 7 months. It could also be calculated by working out the difference between 1 January 2001 and 23 July 2004.

1.5 R6 470,00

1.6 R4 845,07

1.7 Deductions:
(a) Taxes for the Republic of South Africa
(b) Unemployment Insurance Fund
(c) Taxes and UIF. It is law that everyone must pay taxes and contribute to the unemployment fund.

`1.8 Provident fund:
(a) This is a fund to finance her retirement.
(b) R485,25
(c) The company contributes:
12 × R485,25 = R5 823,00
(d) This means that the amount paid into a provident fund is subtracted from her salary before tax is calculated.
(e) There are at least three reasons:
- She wants to be able to finance her retirement
- The company contributes R5 823,00 which she would otherwise not earn.
- It is tax deductible which means that she is saving on tax.

(f) % provident fund:

$$\frac{\text{R485,25}}{\text{R6 470}} \times 100 = 7{,}5\%$$

1.9 Medical aid:
(a) To help finance any medical expenses.
(b) R490,33 × 12 = R5 883,96
(c) % medical aid fund:

$$\frac{\text{R490,33}}{\text{R6 470}} \times 100 = 7{,}6\%$$

1.10 UIF:

(a) % UIF:

$$\frac{R64,70}{R6\,470} \times 100 = 1\%$$

(b) R64,70 × 12 = R776,40

1.11 Company contribution:

(a) Total contribution = R485,25 + R490,33 + R64,70
= R1 040,28

(b) Total per year = R1 040,28 × 12
= R12 483,36

(c) No.

1.12 Taxes:

(a) She pays taxes to the Receiver of Revenue who represents the government.

(b) The money is used to run the country and to pay for education, health, roads etc.

1.13 To work out the amount of tax paid:

(a) 12 × R6 470,00 = R77 640,00

(b) 12 × R485,25 = R5 823,00

(c) R77 640 – R5 823,00 = R71 817,00

(d) The second tax bracket

(e) Tax for the year:

Amount that exceeds R70 000 = R71 817,00 – R70 000
= R1 817,00

$$\text{Tax} = R12\,600,00 + \left[\frac{25}{100} \times R1\,817,000\right]$$

= R13 054,25

(f) Tax rebate for 2004 = R5 400,00

Therefore tax for the year = R13 054,25 – R5 400,00
= R7 654,25

Therefore tax per month = R7 654,25 ÷ 12
= R637,85

(g) Monthly taxable income = R6 470,00 – 485,25
= R5 984,75

From the tables, tax = R595,13

No, it is slightly less.

1.14 After her increase:

(a) **Method 1**

$$\frac{8,5}{100} \times R6\,470,00 = R549,95$$

Therefore new salary = 6 470,00 + R549,95
= R7019,95

Method 2

Her salary will now be 108,5% of the original salary.

$$\frac{108,5}{100} \times R6\,470,00 = R7\,019,95$$

(b) UIF:

$$\frac{1}{100} \times R7019,95 = R70,20$$

(c) Provident fund:

$$\frac{7,5}{100} \times R7019,95 = R526,50$$

(d) Monthly taxable income = R70 19,95 – R526,50
= R6 493,45
Therefore from tables, tax = R707,87

(e) Medical aid:

$\frac{5}{100} \times$ R490,33 = R24,52

Therefore new medical aid = R490,33 + R24,52
= R514,85

(f) Net income = R7 019,95 – (R70,20 + R526,50 + R707,87 + R514,85)
= R5 200,53

Increase in net income = R5 200,53 – R4 845,07
= R355,46

1.15 To work out deductions:
(a) UIF:

$\frac{1}{100} \times$ R10 933 = R109,33
but maximum is set at R88,36 so this is what he pays.

Provident fund:

$\frac{7,5}{100} \times$ R10 933 = R819,98

Monthly taxable income = R10 933 – R819,98
= R10 113,03
Therefore from tables, tax = R1 640,78

EARNINGS		**DEDUCTIONS**	
Basic salary	R10 933,00	Tax RSA	R1 640,78
		Provident fund	R819,98
		UIF	R88,36
		Medical aid	R490,33
		House bond	R2 600,00
TOTAL EARNINGS	R10 933,00	TOTAL DEDUCTIONS	R5 639,45
		NET PAY	R5 293,55

(b) Tax bracket:
12 × R10 933,00 – (12 × R819,98) = R121 356,24
This means he falls into the 3rd tax bracket.

1.16 This favours the people who earn the least amount of money as it will be a greater percentage of their income.

Activity 1 — What am I earning?

At the end of the month each employee receives a pay slip showing how much money he/she has earned over the past month. Each employee will earn a gross amount of money from which deductions will have been made.

Answer the following questions using the information in handout 1.1 — a pay slip for an employee.

1.1 Whose pay slip is this?

1.2 Who does she work for?

1.3 When will her salary be available in the bank?

1.4 How long has this employee been working for this company? Show how you worked this out.

1.5 What is her gross income per month (gross income = total income before deductions)?

1.6 How much of her gross income will actually be deposited into the employee's bank account? This amount is called her net income.

1.7 There are four different amounts that are deducted from this employee's income.
(a) What does Tax RSA mean?
(b) What does UIF stand for?
(c) A compulsory deduction means that she has no choice but to pay this amount. Which of the four deductions are compulsory? Explain why they are compulsory.

1.8 Answer the following questions about the provident fund:
(a) Why does Patricia have a provident fund?
(b) If the company that Patricia works for also contributes to her provident fund by matching her payment, how much does the company contribute per month?
(c) How much does the company contribute per year?
(d) Provident fund contributions are tax deductible. What does that mean?
(e) Explain why is it a good idea to contribute to a provident fund?
(f) What percentage of her gross salary does she contribute to the provident fund?

1.9 Another deduction that appears on Patricia's payslip is medical aid.
(a) Why does she belong to a medical aid fund?
(b) If the company that Patricia works for also contributes to her medical aid fund by matching her payment, how much does the company contribute per year?
(c) What percentage of her gross salary does she contribute to the medical aid fund?

1.10 Answer the following questions on UIF.
(a) What percentage of her gross salary is her UIF?
(b) The company that Patricia works for contributes the same amount to her UIF. How much does the company contribute per year?

1.11 From the previous three questions you can see that the company contributes to three funds on behalf of Patricia.
(a) What is the company's total contribution per month?
(b) What is the company's total contribution per year?
(c) When you look at Patricia's payslip, is it obvious that the company contributes this amount?

1.12 Patricia pays R595,13 tax.
(a) To whom does she pay tax?
(b) What is this money used for?

1.13 Patricia does not pay tax on her gross amount. Her contribution to the provident fund is subtracted first and the remaining amount is taxed. Everyone is taxed according to a particular tax bracket. Look on handout 1.1 for the tax bracket table and use it to answer the following questions.
(a) Patricia earns R6 470 per month. What does she earn per year?
(b) What does she pay for the provident fund per year?
(c) This is tax deductible. What, then, is her taxable income?
(d) Look at the tax bracket table. Into which tax bracket does she fall?
(e) Work out her tax for the year.
(f) Subtract the tax rebate for 2004 (find this at the bottom of the tax bracket table). This is the tax that she must pay for the year. What does she pay per month?
(g) The process followed in (f) is a tedious one, especially if the company employs lots of people. The Receiver of Revenue has monthly tax tables that you can use to work out your tax. Using handout 1.2, look up Patricia's tax. To do this, first work out her monthly taxable income by subtracting her provident fund contribution from her gross income, then look up the resulting amount in the monthly tax table in the handout. Does it agree with what you worked out in (f) ?

1.14 Patricia is told that she is getting an 8,5% increase.
(a) What will her new monthly salary be?
(b) Work out her UIF. UIF is 1% of her gross income up to a maximum of R88,36.
(c) If the provident fund contribution is 7,5% of her gross income what does she pay now?
(d) Using the monthly tax tables on handout 1B, work out her tax per month after her increase.
(e) Medical aid increases by 5%. What will she be paying now?
(f) What will her net income be (net income is the amount that she will actually get in her bank account)?
(g) By how much has her net income increased?

1.15 Patricia's friend, Sizwe, earns R10 933 per month. He has a house bond of R2 600 which is deducted from his salary as well as medical aid which is R490,33 per month.
(a) Draw up a payslip for Sizwe, showing earnings and deductions. Remember that the provident fund is 7,5% of his gross salary and UIF is 1% up to a maximum of R88,36.
(b) Which tax bracket does he fit into?

1.16 Notice that there are 6 tax brackets but everyone gets the same tax rebate. Who does this favour?

PAY SLIP

DLD Payroll Services.

Payslip no.	Employer name	Job title	Employee no.
43	The Finance Institute	Office co-ordinator	FI07 0000392

Employee name	Identity no.	Date engaged
Patricia Prentis	8109270045086	1 January 2001

Date of birth	Sex	Marital status	Deposit institution	Pay date
27 September 1981	F	Not M	Standard Bank	23 July 2004

EARNINGS		DEDUCTIONS	
Basic salary	R6 470,00	Tax RSA	R595,13
		Provident fund	R485,25
		UIF	R64,70
		Medical Aid	R490,33
TOTAL EARNINGS	R6 470,00	TOTAL DEDUCTIONS	R1 624,93
		NET PAY	R4 845,07

TAX BRACKET TABLE FOR QUESTION 1.12

Statutory rates of tax applicable to individuals	
WHERE THE TAXABLE INCOME	
Does not exceed R70 000	18% of each R1 of the taxable income
Exceeds R70 000 but does not exceed R110 000	R12 600 plus 25% of the amount by which the taxable income exceeds R70 000
Exceeds R110 000 but does not exceed R140 000	R22 600 plus 30% of the amount by which the taxable income exceeds R110 000
Exceeds R140 000 but does not exceed R180 000	R31 600 plus 35% of the amount by which the taxable income exceeds R140 000
Exceeds R180 000 but does not exceed R255 000	R45 600 plus 38% of the amount by which the taxable income exceeds R180 000
Exceeds R255 000	R74 100 plus 40% of the amount by which the taxable income exceeds R255 000
Tax rebates for 2004	
Rebates: primary rebate R5 400 -- 65 years and older R3 100	

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MONTHLY DEDUCTION TABLE / MAANDELIKSE AFTREKKINGSTABEL

Remuneration Besoldiging	Annual Equivalent Jaarlikse Ekwivalent	Tax/Belasting	
		Under 65 Onder 65	Over 65 Bo 65
5582 - 5601	67098	523.13	256.47
5602 - 5621	67338	526.73	260.07
5622 - 5641	67578	530.33	263.67
5642 - 5661	67818	533.93	267.27
5662 - 5681	68058	537.53	270.87
5682 - 5701	68298	541.13	274.47
5702 - 5721	68538	544.73	278.07
5722 - 5741	68778	548.33	281.67
5742 - 5761	69018	551.93	285.27
5762 - 5781	69258	555.53	288.87
5782 - 5801	69498	559.13	292.47
5802 - 5821	69738	562.73	296.07
5822 - 5841	69978	566.33	299.67
5842 - 5861	70218	569.93	303.27
5862 - 5881	70458	573.53	306.87
5882 - 5901	70698	577.13	310.47
5902 - 5921	70938	580.73	314.07
5922 - 5941	71178	584.33	317.67
5942 - 5961	71418	587.93	321.27
5962 - 5981	71658	591.53	324.87
5982 - 6001	71898	595.13	328.47
6002 - 6021	72138	598.73	332.07
6022 - 6041	72378	602.33	335.67
6042 - 6061	72618	605.93	339.27
6062 - 6081	72858	609.53	342.87
6082 - 6101	73098	613.13	346.47
6102 - 6121	73338	616.73	350.07
6122 - 6141	73578	620.33	353.67
6142 - 6161	73818	623.93	357.27
6162 - 6181	74058	627.87	361.20
6182 - 6201	74298	632.87	366.20
6202 - 6221	74538	637.87	371.20
6222 - 6241	74778	642.87	376.20
6242 - 6261	75018	647.87	381.20
6262 - 6281	75258	652.87	386.20
6282 - 6301	75498	657.87	391.20
6302 - 6321	75738	662.87	396.20
6322 - 6341	75978	667.87	401.20
6342 - 6361	76218	672.87	406.20
6362 - 6381	76458	677.87	411.20
6382 - 6401	76698	682.87	416.20
6402 - 6421	76938	687.87	421.20
6422 - 6441	77178	692.87	426.20
6442 - 6461	77418	697.87	431.20
6462 - 6481	77658	702.87	436.20
6482 - 6501	77898	707.87	441.20
6502 - 6521	78138	712.87	446.20
6522 - 6541	78378	717.87	451.20
6542 - 6561	78618	722.87	456.20
6562 - 6581	78858	727.87	461.20

Remuneration Besoldiging	Annual Equivalent Jaarlikse Ekwivalent	Tax/Belasting	
		Under 65 Onder 65	Over 65 Bo 65
6582 - 6601	79098	732.87	466.20
6602 - 6621	79338	737.87	471.20
6622 - 6641	79578	742.87	476.20
6642 - 6661	79818	747.87	481.20
6662 - 6681	80058	752.87	486.20
6682 - 6701	80298	757.87	491.20
6702 - 6721	80538	762.87	496.20
6722 - 6741	80778	767.87	501.20
6742 - 6761	81018	772.87	506.20
6762 - 6781	81258	777.87	511.20
6782 - 6801	81498	782.87	516.20
6802 - 6821	81738	787.87	521.20
6822 - 6841	81978	792.87	526.20
6842 - 6861	82218	797.87	531.20
6862 - 6881	82458	802.87	536.20
6882 - 6901	82698	807.87	541.20
6902 - 6921	82938	812.87	546.20
6922 - 6941	83178	817.87	551.20
6942 - 6961	83418	822.87	556.20
6962 - 6981	83658	827.87	561.20
6982 - 7001	83898	832.87	566.20
7002 - 7021	84138	837.87	571.20
7022 - 7041	84378	842.87	576.20
7042 - 7061	84618	847.87	581.20
7062 - 7081	84858	852.87	586.20
7082 - 7101	85098	857.87	591.20
7102 - 7121	85338	862.87	596.20
7122 - 7141	85578	867.87	601.20
7142 - 7161	85818	872.87	606.20
7162 - 7181	86058	877.87	611.20
7182 - 7201	86298	882.87	616.20
7202 - 7221	86538	887.87	621.20
7222 - 7241	86778	892.87	626.20
7242 - 7261	87018	897.87	631.20
7262 - 7281	87258	902.87	636.20
7282 - 7301	87498	907.87	641.20
7302 - 7321	87738	912.87	646.20
7322 - 7341	87978	917.87	651.20
7342 - 7361	88218	922.87	656.20
7362 - 7381	88458	927.87	661.20
7382 - 7401	88698	932.87	666.20
7402 - 7421	88938	937.87	671.20
7422 - 7441	89178	942.87	676.20
7442 - 7461	89418	947.87	681.20
7462 - 7481	89658	952.87	686.20
7482 - 7501	89898	957.87	691.20
7502 - 7521	90138	962.87	696.20
7522 - 7541	90378	967.87	701.20
7542 - 7561	90618	972.87	706.20
7562 - 7581	90858	977.87	711.20

R5582 — R7581 **TABLE / TABEL C**

RIGLYNE VIR WERKGEWERS

SARS

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MONTHLY DEDUCTION TABLE / MAANDELIKSE AFTREKKINGSTABEL

Remuneration Besoldiging	Annual Equivalent Jaarlikse Ekwivalent	Tax/Belasting Under 65 Onder 65	Tax/Belasting Over 65 Bo 65
7582 - 7611	91158	984.12	717.45
7612 - 7641	91518	991.62	724.95
7642 - 7671	91878	999.12	732.45
7672 - 7701	92238	1006.62	739.95
7702 - 7731	92598	1014.12	747.45
7732 - 7761	92958	1021.62	754.95
7762 - 7791	93318	1029.12	762.45
7792 - 7821	93678	1036.62	769.95
7822 - 7851	94038	1044.12	777.45
7852 - 7881	94398	1051.62	784.95
7882 - 7911	94758	1059.12	792.45
7912 - 7941	95118	1066.62	799.95
7942 - 7971	95478	1074.12	807.45
7972 - 8001	95838	1081.62	814.95
8002 - 8031	96198	1089.12	822.45
8032 - 8061	96558	1096.62	829.95
8062 - 8091	96918	1104.12	837.45
8092 - 8121	97278	1111.62	844.95
8122 - 8151	97638	1119.12	852.45
8152 - 8181	97998	1126.62	859.95
8182 - 8211	98358	1134.12	867.45
8212 - 8241	98718	1141.62	874.95
8242 - 8271	99078	1149.12	882.45
8272 - 8301	99438	1156.62	889.95
8302 - 8331	99798	1164.12	897.45
8332 - 8361	100158	1171.62	904.95
8362 - 8391	100518	1179.12	912.45
8392 - 8421	100878	1186.62	919.95
8422 - 8451	101238	1194.12	927.45
8452 - 8481	101598	1201.62	934.95
8482 - 8511	101958	1209.12	942.45
8512 - 8541	102318	1216.62	949.95
8542 - 8571	102678	1224.12	957.45
8572 - 8601	103038	1231.62	964.95
8602 - 8631	103398	1239.12	972.45
8632 - 8661	103758	1246.62	979.95
8662 - 8691	104118	1254.12	987.45
8692 - 8721	104478	1261.62	994.95
8722 - 8751	104838	1269.12	1002.45
8752 - 8781	105198	1276.62	1009.95
8782 - 8811	105558	1284.12	1017.45
8812 - 8841	105918	1291.62	1024.95
8842 - 8871	106278	1299.12	1032.45
8872 - 8901	106638	1306.62	1039.95
8902 - 8931	106998	1314.12	1047.45
8932 - 8961	107358	1321.62	1054.95
8962 - 8991	107718	1329.12	1062.45
8992 - 9021	108078	1336.62	1069.95
9022 - 9051	108438	1344.12	1077.45
9052 - 9081	108798	1351.62	1084.95

Remuneration Besoldiging	Annual Equivalent Jaarlikse Ekwivalent	Tax/Belasting Under 65 Onder 65	Tax/Belasting Over 65 Bo 65
9082 - 9111	109158	1359.12	1092.45
9112 - 9141	109518	1366.62	1099.95
9142 - 9171	109878	1374.12	1107.45
9172 - 9201	110238	1381.62	1114.95
9202 - 9231	110598	1389.12	1122.45
9232 - 9261	110958	1396.62	1129.95
9262 - 9291	111318	1404.12	1137.45
9292 - 9321	111678	1411.62	1144.95
9322 - 9351	112038	1419.12	1152.45
9352 - 9381	112398	1426.62	1159.95
9382 - 9411	112758	1434.12	1167.45
9412 - 9441	113118	1441.62	1174.95
9442 - 9471	113478	1449.12	1182.45
9472 - 9501	113838	1456.62	1189.95
9502 - 9531	114198	1464.12	1197.45
9532 - 9561	114558	1471.62	1204.95
9562 - 9591	114918	1479.12	1212.45
9592 - 9621	115278	1487.78	1221.11
9622 - 9651	115638	1496.78	1230.11
9652 - 9681	115998	1505.78	1239.11
9682 - 9711	116358	1514.78	1248.11
9712 - 9741	116718	1523.78	1257.11
9742 - 9771	117078	1532.78	1266.11
9772 - 9801	117438	1541.78	1275.11
9802 - 9831	117798	1550.78	1284.11
9832 - 9861	118158	1559.78	1293.11
9862 - 9891	118518	1568.78	1302.11
9892 - 9921	118878	1577.78	1311.11
9922 - 9951	119238	1586.78	1320.11
9952 - 9981	119598	1595.78	1329.11
9982 - 10011	119958	1604.78	1338.11
10012 - 10041	120318	1613.78	1347.11
10042 - 10071	120678	1622.78	1356.11
10072 - 10101	121038	1631.78	1365.11
10102 - 10131	121398	1640.78	1374.11
10132 - 10161	121758	1649.78	1383.11
10162 - 10191	122118	1658.78	1392.11
10192 - 10221	122478	1667.78	1401.11
10222 - 10251	122838	1676.78	1410.11
10252 - 10281	123198	1685.78	1419.11
10282 - 10311	123558	1694.78	1428.11
10312 - 10341	123918	1703.78	1437.11
10342 - 10371	124278	1712.78	1446.11
10372 - 10401	124638	1721.78	1455.11
10402 - 10431	124998	1730.78	1464.11
10432 - 10461	125358	1739.78	1473.11
10462 - 10491	125718	1748.78	1482.11
10492 - 10521	126078	1757.78	1491.11
10522 - 10551	126438	1766.78	1500.11
10552 - 10581	126798	1775.78	1509.11

R7582 — R10581 TABLE / TABEL C

SARS *GUIDELINES FOR EMPLOYERS*

Activity 2 — Budgeting

This activity follows on from activity 1. The students have looked at what they will actually receive in their bank account and will now look at how to budget .

ABOUT THIS ACTIVITY

This activity focuses on the types of expenses that one incurs during a month and helps to categorise these expenses into essential and non-essential expenses. The aim is to help the students formulate a method of budgeting. This activity addresses all the AC of SO 1 of the personal finance unit.

MANAGING THIS ACTIVITY

This activity has a worksheet and one handout. This activity could be extended to allow the students to do their own personal budget.

2.1

Fixed expenses	Changing expenses
Rent	Water
Household insurance	Clothes
gym	Chemist
Alarm company	Telephone
Car repayments	Petrol
	Food
	Car repairs
	Electricity
	Bus fare

2.2 Irregular expenses:

(a) Car service per month:

$$\frac{R1000}{12} = R83,33$$

(b) TV License:

$$\frac{R208}{12} = R17,33$$

(c) Car License:

$$\frac{R138}{12} = R11,50$$

(d) House maintenance:

$$\frac{R2000}{12} = R166,67$$

(e) Doctor's bills:

$$\frac{R600}{12} = R50$$

2.3 Patricia's expenses:

(a) Classification:

Fixed expenses	Changeable expenses	Irregular expenses
rent armed response household insurance telephone rental unit trust charities car repayment gym parking car insurance	electricity water pharmacy cell phone landline calls entertainment (movies, concert) take aways gifts petrol groceries clothes weekend away doctor haircut restaurant lamp	TV licence car tyres car service annual holiday

(b) Average for a month:

Fixed expenses:	**Average per month**
Rent	R813,00
Armed response	R34,50
Household insurance	R64,25
Telephone rental	R19,21
Unit trust	R250,00
Charities	R75,00
Car repayments	R750,00
Gym	R95,00
Parking	R125,00
Car insurance	R78,00
Sub-total: Fixed expenses	**R2 303,96**
Changing expenses:	
Electricity	R41,30
water	R12,31
Pharmacy	R113,67
Cell phone	R246,09
Landline calls	R53,44
Movies	R55,00
Concert	R25,00
Take aways	R45,50
Gifts	R41,67
Petrol	R266,00
Groceries	R627,90
Clothes	R340,43
Weekend away	R190,00
Doctor	R40,00
Restaurant	R35,00
lamp	R48,33
Haircut	R56
Sub-total: changing expenses	**R2 237,64**

Savings for irregular expenses:		
	Annual	
TV licence	R208	R17,33
Car tyres	R2 400	R200
Car service	R950,00	R79,17
Annual holiday	R5 500,00	R458,33
Sub-total: saving for irregular expenses		**R754,66**
TOTAL EXPENSES AND SAVINGS		**R5 296,43**

(c) Difference between income and expenditure = R4 845,07 – R5 296,43
= –R451,36

2.4 This might differ from student to student.

Essential fixed expenses	**Savings and investments**	**Essential changeable expenses**	**Non-essential expenses**
Rent	Unit trusts	Electricity	Charities
Armed response		Water	Landline calls
Household insurance		Pharmacy	Movies
Telephone rental		Gifts	Take aways
Car repayments		Petrol	Gym
Car insurance		Groceries	Holiday
TV licence		Clothes	Weekend away
Parking		Doctor	Restaurant
		Haircut	Concert
		Car tyres	Lamp
		Car service	

2.5 Total is about R1 137

$$\text{Percentage of income} = \frac{\text{R1 137}}{\text{R4 845,07}} \times 100$$
$$= 23{,}5\%$$

2.6 This will differ from student to student. Try encourage them to be realistic.

2.7 Savings

(a) 10% of net income:

$$\frac{10}{100} \times \text{R4 845,07} = \text{R484,51}$$

(b) Patricia is saving R250 in unit trust:

$$\frac{250}{4\,845{,}07} \times \text{R100} = 5{,}16\%$$

(c) This will differ from student to student.

2.8 Suggestions like school fees, housing bond, domestic worker wages etc.

Activity 2 — Am I spending too much?

In the previous activity you looked at a pay slip and saw how what you think you earn differs from what you actually receive in your bank account. In this activity you will look at how to budget for the month so as not to overspend and get into debt. We will continue to use Patricia as an example.
There are two types of expenses every month that must be taken into consideration. The first type are called fixed expenses as they remain the same every month and the second type are called changing expenses.

2.1 Draw up a table with two columns, one with the heading fixed expenses and the other with the heading changeable expenses. Classify the following expenses as fixed or changeable. Fill them into your table.

Rent	Gym	Food
Water	Telephone	Car repayments
Household insurance	Chemist	Car repairs
Clothes	Alarm company	Electricity
Chemist	Petrol	Bus fare

2.2 There are also expenses which we will call irregular expenses as they are paid only once a year. For each of the following irregular expenses work out how much it would cost per month if you were to pay a portion of each expense monthly:
(a) Car service @ R1 000
(b) TV licence @ R208
(c) Car licence @ R138
(d) House maintenance @ R2 000
(e) Doctor's bills @ R600

2.3 Patricia wants to budget to make sure that she doesn't overspend. She shares some of the household expenses with her housemates. She keeps a record of her expenses for three months (see Handout 2). Use the information on the handout to do the following tasks.
(a) Classify each expense as one of the following; fixed expense, changeable expense, irregular expense.
(b) Redraw the table. Work out on average what Patricia spends per month on each of the expenses and fill these averages in the appropriate spaces. You might need to leave more lines per category. Calculate Patricia's total average monthly expenses.

A look at Patricia's month	
Fixed expenses:	**Average per month**
Sub-total: Fixed expenses	
Changing expenses:	
Sub-total: changing expenses	

Savings for irregular expenses:		
	Annual	
Sub-total: saving for irregular expenses		
TOTAL EXPENSES AND SAVINGS		

(c) From activity 1 we know that Patricia earns R4 845,07 net income. Find the difference between her income and her expenses.

2.4 From question 2.3 one can see that Patricia is spending more than she is getting in. She would like to get this under control as well as save towards a deposit for a house. Help Patricia to get an idea of what she is spending her money on by classifying her expenses into the four categories shown in the table.

Essential fixed expenses	Savings and investments	Essential changing expenses	Non-essential expenses

2.5 Total all her monthly non-essential expenses. What percentage of Paticia's income is she spending on non-essential expenses?

2.6 Go through the list of expenses and suggest ways in which Patricia could save more each month. Be realistic about your suggestions and give reasons why you think each would work. Work out approximately how much she would save.

2.7 Economists say that you should be saving between 10% and 15% of your net income (income after all your deductions).
(a) Calculate 10% of Patricia's net income.
(b) What percentage of Paticia's net income is she saving before you change her spending?
(c) What percentage will she save if she listens to your suggestions?

2.8 The above exercise is a way in which you can get a picture of what your expenses each month are, as well as predicting how much you will need to earn in order to maintain that lifestyle. Can you think of other expenses that Patricia doesn't have but that could occur?

Notes made by Patricia

March
- rent = R813
- electricity = R45
- water = R13
- armed response = R34,50
- household insurance = R64,25
- telephone rental = R19,21
- TV licence =R208 for the year
- car tyres = R2 400 (replace annually)
- pharmacy = R177,84
- cell phone = R245,36
- landline calls = R48,57
- Unit trusts = R250
- charities R75
- car repayment = R750
- movies R33 per movie(went twice)
- take aways = R56,50
- Mom's birthday = R125
- gym = R95
- petrol = R250
- parking = R125
- groceries = R667,87
- clothes R545,60

April
- rent = R813
- electricity = R37
- water = R14,50
- armed response = R34,50
- household insurance = R64,25
- telephone rental = R19,21
- weekend away = R570
- pharmacy = R135,68
- cell phone = R260,46
- landline calls = R53,78
- Unit trusts = R250
- charities R75
- car repayment = R750
- movies R33 per movie(went once)
- take aways = R80
- doctor R120,00
- gym = R95
- petrol = R312
- parking = R125
- groceries = R567,87
- clothes R350
- car insurance = R78
- haircut = R168

May
- rent = R813
- electricity = R41,90
- water = R9,45
- armed response = R34,50
- household insurance = R64,25
- telephone rental = R19,21
- annual car service = R950
- pharmacy = R27,50
- cell phone = R232,45
- landline calls = R57,98
- Unit trusts = R250
- charities R75
- car repayment = R750
- movies R33 per movie(went twice)
- restaurant = R105
- lamp = R145
- gym = R95
- petrol = R236
- parking = R125
- groceries = R647,97
- clothes R125,68
- car insurance = R78
- concert = R75
- annual holiday = R5 500

Activity 3 — Inflation

One of the problems with budgeting is that the price of commodities fluctuates so much that one's projected budget is not always accurate. This activity explores this change in prices.

ABOUT THIS ACTIVITY
This activity looks at price increases over the past 10 years. The concepts of inflation and financial indicators are introduced and inflation is related to the previous activities. This activity addresses AC2, 3 and 4 of SO2 of the personal finance unit standard.

MANAGING THIS ACTIVITY
This activity requires the students to interview their parents and grandparents so this needs to be done the day before as a homework task. The rest of the worksheet requires knowledge of percentages. A calculator is needed. An Excel spreadsheet could be used to do repetitive calculations.

3.1 Individual responses.

3.2 Individual responses.

3.3 Individual responses.

3.4 Individual responses.

3.5 Increase in prices over 10 years:

Item	1995	2004	Increase in price	% increase	Average increase per year
Brown sliced bread	R1,78	R4,15	R2,37	$\frac{2,37}{1,78} \times 100 = 133,2\%$	$\frac{133,2}{10} = 13,3\%$
Mielie meal (1kg)	R1,94	R3,99	R2,05	$\frac{2,05}{1,94} \times 100 = 105,7\%$	$\frac{105,7}{10} = 10,6\%$
Sugar (2,5kg)	R6,76	R11,99	R5,23	$\frac{5,23}{6,76} \times 100 = 77,4\%$	$\frac{77,4}{10} = 7,7\%$
Milk (1litre)	R2,09	R5,69	R3,60	$\frac{3,60}{2,09} \times 100 = 172,2\%$	$\frac{172,2}{10} = 17,2\%$
Minced meat (per kg)	R16,40	R28,95	R12,55	$\frac{12,55}{16,40} \times 100 = 76,5\%$	$\frac{76,5}{10} = 7.7\%$
Rice (2kg)	R5,57	R11,99	R6,42	$\frac{6,42}{5,57} \times 100 = 115,3\%$	$\frac{115,3}{10} = 11,5\%$
Margarine (500 kg)	R3,67	R8,49	R4,82	$\frac{4,82}{3,67} \times 100 = 131,3\%$	$\frac{131,3}{10} = 13,1\%$
Potatoes (10 kg)	R12,58	R32,99	R20,41	$\frac{20,41}{12,58} \times 100 = 162,2\%$	$\frac{16,2}{10} = 16,2\%$
Candles (packet of 6)	R2,17	R6,39	R4,22	$\frac{4,22}{2,17} \times 100 = 194,5\%$	$\frac{194,5}{10} = 19,4\%$
Petrol (leaded per litre)	R1,74	R4,17	R2,43	$\frac{2,43}{1,74} \times 100 = 139,7\%$	$\frac{139,7}{10} = 13,9\%$

3.6 Candles increased the most, possibly as they are made of paraffin which is a fuel.

3.7 Sugar and minced meat increased the least.

3.8 Average = (13,3 + 10,6 + 7,7 + 17,2 + 7,7 + 11,5 + 13,1 + 16,2 + 19,4 + 13,9) ÷ 10 = 13,1%

3.9 It means that her salary increase is less than the inflation rate. This means that her buying power has decreased. She will have to look carefully at her budget to ensure that she keeps within her means.

3.10 House increase:

(a) R125 000 + R65 000 = R190 000

(b) Percentage increase:

$$\frac{950\,000 - 190\,000}{190\,000} \times 100 = 400\%$$

(c) Average increase over 13 years:

$$\frac{400}{13} = 30{,}76\%$$

(d) The demand for housing.

(e) Houses appreciate in value often at a rate above inflation. When you put money in the bank, the interest you earn is linked to inflation so buying property helps you save/earn more than you would if you put the money in the bank.

Activity 3 — Inflation

In the last two activities you looked at Patricia's salary and then at how she spent that salary. In this activity you will look at prices of goods and how they have changed over the years.

3.1 Take some time and interview someone who has been purchasing items for more 30 years (eg your parents or grandparents). Ask them if they can remember how much things cost when they were teenagers and young adults. Try get a range of items for example bread, milk, Coke, cigarettes, petrol, newspaper, movies, shoes, cars, houses, sweets.

3.2 Once you have an idea from the older generation as to what things used to cost, see if you can find out how much those same items cost now.

3.3 Find out what choice of products there was on the market. For example breakfast cereal, sweets, chocolates, cooldrink, cars. Compare these to the number of choices that there are today.

3.4 Try to get an idea of how much they earned when they started work and how much a person in that job would earn now.

3.5 When one speaks to an older generation one realises how the economy has changed over the years. Prices seemed to have increased dramatically, as have salaries. This phenomenon is called inflation and the study of it keeps our country's economists very busy.

The following list shows how 10 basic commodities have increased in price over a 10 year period. Redraw the table and fill in the blank spaces. The first example has been done for you.

Item	1995	2004	Increase in price	% increase	Average increase per year
Brown sliced bread	R1,78	R4,15	R2,37	$\frac{2,37}{1,78} \times 100 = 133,2\%$	$\frac{133,2}{10} = 13,3\%$
Mielie meal (1kg)	R1,94	R3,99	R		
Sugar (2,5kg)	R6,76	R11,99	R		
Milk (1litre)	R2,09	R5,69	R		
Minced meat (per kg)	R16,40	R28,95	R		
Rice (2kg)	R5,57	R11,99	R		
Margarine (500 kg)	R3,67	R8,49	R		
Potatoes (10 kg)	R12,58	R32,99	R		
Candles (packet of 6)	R2,17	R6,39	R		
Petrol (leaded per litre)	R1,74	R4,17	R		

3.6 Notice that all the items did not increase by the same percentage. Which item increased the most? Can you think of a reason for this?

3.7 Which item increased the least?

3.8 What is the average increase per year of all 10 of the commodities?
Economists do calculations like this but on a much larger scale with many more commodities, and come up with a figure that gives one an idea of what the inflation rate is. This figure is called the Consumer Price Index (CPI). This number includes bond repayments (money borrowed from the bank to buy houses) so a more useful figure is the CPIX which excludes bond repayments. The CPIX shows how food, clothes, household goods and transport increase in price over a period of time.

3.9 In activities 1 and 2 we looked at Patricia's income and expenditure. She was earning R6 470,00 per month and then received an 8,5% increase which means she is now earning R7 019,95. What will it mean for Patricia if the CPIX is set at 11,2%?

3.10 The Donaldson family bought a house in Claremont in 1991 for R125 000. They did renovations to the amount of R65 000. In 2004 it was valued at R950 000.
(a) How much did they pay for the house after the renovation?
(b) What is the percentage increase over the 13 years?
(c) What is the average increase per year?
(d) This is much higher than the increase which you calculated for the commodities in question 3.5. What other factors, besides inflation, would drive the house prices to increase so much?
(e) Why do you think it is a good idea to buy property?

Activity 4 — Contract or prepaid?

One of the difficulties of managing money these days is the amazing amount of choice that one has as a consumer. Any purchase requires research. When choosing a cell phone contract one needs to make many choices and do many calculation to make sure one gets the most economical package that suits one's needs.

ABOUT THIS ACTIVITY

In this activity the students compare the benefits of a cell phone contract and a pay-as-you-go package. They will have to read information off an advertisement and justify why they choose certain contracts. This activity addresses AC 2 and 4 of the SO1 of the personal finance unit standard.

MANAGING THIS ACTIVITY

Students need the worksheet for this activity as well as the handout which has all the options for them to work with. This activity could be done in groups of two as it lends itself to discussion. Since answers will vary students must be able to justify their responses.

4.1 These terms might need to be discussed as the information provided only gives actual times.
(a) Peak times are when most calls are made, usually during business hours.
(b) Off-peak times are when fewer calls are made, usually after business hours.

4.2 Off-peak times : 20:00-07:00 Mon to Fri, weekends & public holidays.
Peak times : 07:00-20:00 Mon to Fri. Calls in peak times are much more expensive than calls in off-peak times.

4.3 Again, these might need to be discussed. A contract is an agreement between the provider and the user. In the case of a cell phone contract the provider charges a monthly subscription fee for which they may give some free minutes and other benefits. In addition to the subscription fee they also invoice the user for time spent on calls. The pre-paid minutes may typically be more expensive than the contract minutes.

4.4 Price of SMS:
(a) 34c
(b) 80c
(c) 34c
(d) 80c
(e) Price of 1 SMS for 100 bundle:
$\frac{32}{100} = R0,32$
(f) Price of 1 SMS for 200 bundle:
$\frac{44}{200} = R0,22$
$= 22c$

(g) Price of 1 SMS for 500 bundle:
$\frac{110}{500} = R0,22$
$= 22c$

4.5 Only if you make more than 100 SMS's per month.

4.6 Basic monthly charge:
(a) Peter:

Weekend/everyday	R135,00
Itemised billing	R17,10
Lost card protection	R5,70
TOTAL	**R157,80**

(b) Zainub:

Talk 120	R315,00
200 bundle	R44,00
Itemised billing	R17,10
SMS balance notification	R6,50
TOTAL	**R382,60**

(c) Thembi:

Business	R180,00
100 bundle	R32,00
Emergency-147	R11,40
Insurance band 1	R39,00
TOTAL	**R262,40**

(d) Walter:

Family Top-up	R135,00
200 bundle	R44,00
Weekend weather subscription	R10,00
TOTAL	**R189,00**

4.7 Per second and per minute billing:

(a) 1min 45s:

Per second	Per minute
(60 + 45) × 6c = R6,30	This will be billed as 2 minutes: 2 × R3,60 = R7,20

(b) 35s:

Per second	Per minute
35 × 6c = R2,10	This will be billed as 1 minute: 1 × R3,60 = R3,60

(c) 2min 03s :

Per second	Per minute
(2 × 60 + 3) × 6c = R7,38	This will be billed as 2½ minutes: 2½ × R3,60 = R9,00

(d) 2min 29s :

Per second	Per minute
(2 × 60 + 29) × 6c = R8,94	This will be billed as 2½ minutes: 2½ × R3,60 = R9,00

(e) The per second billing.

(f) The last two examples show that if you speak for 1 second more than a full minute or half minute you pay for a half minute, so to keep costs down you must try to keep within the full minute or half minute.

4.8 Accounts: This is quite a complicated calculation as you must take into account which contract the person is on, how many and when their free minutes are, whether it is peak or off-peak time, to which provider they are phoning as well as the type of billing.

(a) Siyabonga:

Week/day contract				R135,00
	Peak/off-peak	**Tariff**	**Calculation**	
Call 1	off-peak			free time
Call 2	off-peak			free time
Call 3	peak	R2,70	1½ × R2,70	R4,05
Call 4	peak	80c		R0,80
Call 5	peak	R1,80	1 × R1,80	R1,80
Call 6	peak	R2,75	3 × R2,75	R8,25
Call 7	peak	R2,70	3 × R2,70	R8,10
Call 8	peak	R2,70	2 × R2,70	R5,40
Call 9	peak	R1,80	½ × R1,80	R0,90
Call 10	off-peak			free time
Call 11	off-peak			free time
Call 12	off-peak			free time
Call 13	off-peak			free time
TOTAL				**R164,30**

(b) Feryal :

			Vodago standard	R00,00
	Peak/off-peak	**Tariff**	**Calculation**	
Call 1	off-peak	R1,55	4 × R1,55	R6,20
Call 2	off-peak	R1,55	3 × R1,55	R4,65
Call 3	peak	R2,85	1½ × R2,85	R4,28
Call 4	peak	80c		R0,80
Call 5	peak	R2,85	1 × R2,85	R2,85
Call 6	peak	R2,85	3 × R2,85	R8,55
Call 7	peak	R2,85	3 × R2,85	R8,55
Call 8	peak	R2,85	2 × R2,85	R5,70
Call 9	peak	R2,85	½ × R2,85	R1,43
Call 10	off-peak	R1,55	3½ × R1,55	R5,43
Call 11	off-peak	R1,55	1 × R1,55	R1,55
Call 12	off-peak	R1,55	1 × R1,55	R1,55
Call 13	off-peak	R1,55	9 × R1,55	R13,95
TOTAL				**R65,49**

(c) Biren :

40				R0,00
	Peak/off-peak	**Tariff**	**Calculation**	
Call 1	off-peak	99c	4 × R0,99	R3,96
Call 2	off-peak	1,8c per sec	(2 × 60 + 6) × 18c	R2,27
Call 3	peak	R2,70	1½ × R2,70	R4,05
Call 4	peak	80c		R0,80
Call 5	peak	6c per sec	45 × 6c	R2,70
Call 6	peak	R3,60	3 × R3,60	R10,80
Call 7	peak	R3,60	3 × R3,60	R10,80
Call 8	peak	R3,60	2 × R3,60	R7,20
Call 9	peak	6c per sec	23 × 6c	R1,38
Call 10	off-peak	99c	3½ × R0,99	R3,47
Call 11	off-peak	1,8c per sec	60 × 1,8c	R1,08
Call 12	off-peak	1,8c per sec	32 × 1,8c	R0,58
Call 13	off-peak	1,8c per sec	(8 × 60 + 36) × 1,8c	R9,29
TOTAL				**R58,38**

4.9 Biren did. Peter would need to make more of his calls during off-peak time. If he also has a landline he should try use his cell phone for all his off-peak calls so as to save on his regular phone bill.

4.10 The Talk 120 option benefits the user who needs to make calls during peak hours. This is done in two ways. Firstly, the free time is not restricted to off-peak hours as is the weekend/everyday contract. Secondly, the peak tariffs are cheaper, especially the calls to Telkom.

4.11 The weekend/everyday contract has 120 free minutes whereas the Family Top-up has R135 talktime. The advantage of the Top-up is that the talktime can be used for peak or off-peak time whereas with the weekend/everyday contract the free time is only for off-peak periods. If the user of the Top-up only phones in off-peak time, then the number of minutes they have available will be:

$\frac{135}{0,90}$ = 150 minutes which is 30 minutes more than the weekend/everyday contract.

4.12 This question is quite open-ended and could be a good source of discussion.

(a) Warwick's account is less than R100 so he should take a prepaid contract. He mainly needs off-peak time so the 40 would be a good option as the off-peak tariffs are very low. A 100 SMS bundle may be advisable.
(b) Mrs Fourie needs a prepaid. A Vodago standard is possibly the best as she would probably phone in peak time. An Emergency -147 plus would be advised.
(c) Dylan's Garden Services needs a business contract with an everyday weather subscription.
(d) Hoosaine needs a Talk 120 or 240 with one of the insurance bands and itemised billing.
(e) Viwe needs Vodago prepaid.

Activity 4 — Contract or prepaid

Part of staying within your budget is making sure that you always get the best deal when purchasing something. Deciding on what cell phone contract to purchase is a difficult decision as the choice of contracts is so great. The handout provided shows an advertisement given out by a cellular network. Its aim is to help you choose the best deal for your needs. Answer the following questions about cell phone contract using the handout.

4.1 What is meant by the terms:
(a) Peak times?
(b) Off-peak times?

4.2 What are these times for this network provider and how do these times impact the prices?

4.3 What is meant by the term 'contract' as opposed to 'prepaid'?

4.4 What is the price per SMS in each of the following cases:
(a) Contract off-peak time?
(b) Contract peak time?
(c) Prepaid peak time?
(d) Prepaid off-peak time?
(e) 100 SMS bundle?
(f) 200 SMS bundle?
(g) 500 SMS bundle?

4.5 When would it be advisable to take an SMS bundle?

4.6 Work out the basic monthly charge for the following people. They have already paid their connection fee.
(a) Peter wants a weekend/everyday contract with itemised billing and a lost card protection option.
(b) Zainub wants a Talk 120 contract, a 200 SMS bundle, itemised billing and SMS balance notification.
(c) Thembi wants a business contract, 100 SMS bundle, Emergency-147 plus and Insurance band 1.
(d) The surfer Walter goes prepaid. He wants a Family Top-up, 200 SMS bundle and weekend weather subscription.

4.7 There are different types of billing. Per second billing means that you pay a set rate for every second that you are connected. Per minute billing can mean that you pay a certain rate for every half minute(30 seconds) or part thereof that you are connected. For example : Let us say that you have been connected for 22 seconds. The per second billing rate is 6c per sec and the per minute billing is R3,60 per min. You would pay $22 \times 6 =$ R1,32 for per second billing and for per minute billing you would pay for half a minute which would work out to R3,60 ÷ 2 = R1,80. Do the following examples using the rates given in the example above.
(a) What would you pay for a call of 1min 45s for each type of billing?
(b) What would you pay for a call of 35s for each type of billing?
(c) What would you pay for a call of 2min 03s for each type of billing?
(d) What would you pay for a call of 2min 29s for each type of billing?
(e) Which billing is more cost effective (cheaper for you)?
(f) If your phone is set for per minute billing, suggest ways to keep the cost of calls down.

4.8 Work out what the following people would pay for this account:

(a) Siyabonga has a weekend/everyday contract and makes the following calls:

Call to	Day	Time of call	Length of call
Vodacom	Sunday	18:13	03:56
MTN	Tuesday	20:00	2:06
Telkom	Monday	16:04	01:05
SMS	Monday	11:30	-
Vodacom	Friday	14:30	00:45
MTN	Friday	10:30	02:50
Telkom	Wednesday	15:09	02:32
Telkom	Wednesday	10:34	01:55
Vodacom	Thursday	11:52	00:23
Telkom	Saturday	9:00	03:29
MTN	Saturday	9:05	01:00
MTN	Saturday	9:12	00:32
MTN	Saturday	12:58	08:36

(b) If Feryal has Vodago Standard and makes the same calls as Siyabonga, what would she pay?

(c) If Biren has a 40 prepaid deal and makes the same calls as Siyabonga, what would he pay?

4.9 Which of the above - Peter, Feryal or Biren - benefited most from his/her choice of cell phone package? Suggest ways in which Peter could use his package to benefit him more.

4.10 The weekend/everyday contract and the Talk 120 both give 120 free minutes, yet the monthly charge for the Talk 120 is so much higher. Compare these two contracts and make suggestions as to who would benefit from each.

4.11 The weekend/everyday contract and the Family Top-up prepaid appear very similar. What is the difference between the two contracts? If you had a Top-up prepaid and only phoned in off-peak time, how many minutes would you have available?

4.12 Pretend that you are a Vodacom sales person. With the help of the 3 questions given in the advertisement, advise the following people as to what contract or prepaid they should purchase and what value added services they might enjoy.

(a) Warwick is 18 years old and calls his girlfriend every second evening and on the weekends. He also uses a lot of SMS's. His account is not more than R100 a month.

(b) Mrs Fourie is a pensioner and phones very occasionally. She does need her phone for emergencies though.

(c) Dylan's Garden Services needs a weather report daily, but uses less than 30 minutes a week actual call time. His monthly bill is over R100.

(d) Hoosaine is a high-flying young executive who needs lots of talk time all the time. He travels a lot so needs good insurance. His company requires him to have itemised billing.

(e) Viwe spends on average under R100 per month on calls but he is captain of the soccer team so needs talk time at all times of the day.

The following extract is taken from a service provider's advertisement

CHOOSE THE RIGHT PACKAGE

Question 1: How much will you spend on calls a month?

Under R100
Prepaid is a cheaper option for you

Over R100
Contract is a cheaper option for you

Question 2: When do you make most of your calls?

Evening Weekend Day

(Anytime)

Evening Weekend

(Off-Peak)
Weekend/Everyday

Question 3: How much time do you spend making calls

Over 30 mins a week → Talk contract

Under 30 mins a week → Business contract

Off-peak times: 20:00-07:00 Mon to Fri, weekends & public holidays Peak times: 07:00-20:00 Mon to Fri

CONTRACT	Weekend/everyday	Business	Talk 120	Talk 240	Talk 400 S
Connection fee	R97.00	R97.00	R97.00	R97.00	R97.00
Monthly charge	R135.00	R180.00	R315.00	R430.00	R775.00
Free minutes per month	120(off-peak)	-	120	240	500
Peak Vodacom to Vodacom	R1.80	R1.76	R1.72	R1.72	R1.50p/min
Peak Vodacom to other providers	R2.75	R2.30	R2.30	R2.30	R1.74p/min
Off-peak national charges	90c	90c	90c	90c	90c
Peak standard calls to Telkom	R 2.70	R 1.76	R 1.72	R 1.71	R1.35p/min
Off-peak SMS message	34c	34c	34c	34c	34c
Peak SMS message	80c	80c	80c	80c	80c

PREPAID	40	Vodago standard	Vodago Smartstep	Family Top-up	Business Top-up	SMS BUNDLES	
Connection fee	R199	R199	-	R97	R97	SMS's	R/per month
Monthly charge	-	-	-	R135	R315	100	R32
Inclusive airtime	-	-	-	R135	R315	200	R44
Peak Vodacom to Vodacom	R3.60(6c/sec)	R2.85	-	R1.80	R1.72	500	R110
Peak Vodacom to other providers	R3.60	R2.85	-	R2.75	R2.30	1000	R220
Off-peak national charges	99c	R1.55	-	90c	90c	1500	R330
Peak standard calls to Telkom	R3.60	R2.85	-	R2.70	R1.72	2000	R440
Off-peak SMS message	34c	34c	34c	34c	34c	Talk 500 includes 100 free SMS per month. Talk1000 includes 200 per month	
Peak SMS message	80c	80c	80c	80c	80c		

• *Off peak MTN /Cell-C are charged at 1.8cents p/sec and R1.08 p/min*

VALUE ADDED SERVICES	PRICE PER MONTH
QuickSIM	R3.00
Lost card protection	R5.70
SMS balance notification	R6.50
Call line identity presentation	R9.50
Itemised billing	R17.10
Weather subscription every day	R13.00
Weather subscription weekend	R10.00
Emergency-147 plus	R11.40
Emergency-147 peace of mind	R29.50
Insurance Band 1(cover up to R1000)	R39.00
Insurance Band 2(cover up to R2500)	R49.00
Insurance Band 3(cover up to R5000)	R79.00
Insurance Band 4(cover up to R7500)	R109.00
Insurance Band 5(cover up to R12500)	R149.00

Activity 5 — All about interest.

This activity looks at interest and growth of investments.

ABOUT THIS ACTIVITY

The aim of this activity is to look at capital growth. In this activity the students look at three different scenarios pertaining to interest. In the first scenario the interest is not added to the principal amount. In the second scenario interest is compounded monthly and in the third scenario some of the interest is added to the principal. This activity addresses all the assessment criteria of SO2 of the personal finance unit standard.

MANAGING THIS ACTIVITY

This activity consists of a worksheet only. Students will need a calculator or they can do the calculations using an Excel spreadsheet. A knowledge of percentages is required. The calculations of compound interest can be done in ways other than those used in the activity. These are shown in the answers. The facilitator can choose to expand on this.

5.1 Interest rate per annum = 8,5%

Therefore interest rate per month = $\frac{8,5}{12}$

= 0,7083%

5.2 Every month Mr Mahou would receive:

$\frac{0,7083}{100} \times$ R90 000 = R637,50

5.3 Total amount earned in one year:

5.4 No, he still had only R90 000 in the bank at the end of the year.

5.5

Original amount	R90 000
% interest earned per month	8,5% ÷ 12 = 0,7083%
Interest earned after the first month	$\frac{0,7083}{100} \times$ R90 000 = R637,50
Amount at the end of the first month	R90 000 + R637,50 = R90 637,50
Interest earned after the second month	$\frac{0,7083}{100} \times$ R90 637,50 = R641,99
Amount at the end of the second month	R90 637,50 + R641,99 = R91 279,49
Interest earned after the third month	$\frac{0,7083}{100} \times$ R91 279,49 = R646,53
Amount at the end of the third month	R91 279,49 + R646,53 = R91 926,02
Interest earned after the fourth month	$\frac{0,7083}{100} \times$ R91 926,02 = R651,11
Amount at the end of the fourth month	R91 926,02 + R651,11 = R92 577,13
Interest earned after the fifth month	$\frac{0,7083}{100} \times$ R92 577,13 = R655,72
Amount at the end of the fifth month	R92 577,13 + R655,72 = R93 232,85
Interest earned after the sixth month	$\frac{0,7083}{100} \times$ R93 232,85 = R660,37
Amount at the end of the sixth month	R93 232,85 + R660,37 = R93 893,22
Interest earned after the seventh month	$\frac{0,7083}{100} \times$ R93 893,22 = R665,05

Amount at the end of the seventh month	R93 893,22 + R665,05 = R94 558,27
Interest earned after the eighth month	$\frac{0,7083}{100} \times$ R94 558,27 = R669,76
Amount at the end of the eighth month	R94 558,27 + R669,76 = R95 228,03
Interest earned after the ninth month	$\frac{0,7083}{100} \times$ R95 228,03 = R674,50
Amount at the end of the ninth month	R95 228,03 + R674,50 = R95 902,53
Interest earned after the tenth month	$\frac{0,7083}{100} \times$ R95 902,53 = R679,28
Amount at the end of the tenth month	R95 902,53 + R679,28 = R96 581,81
Interest earned after the eleventh month	$\frac{0,7083}{100} \times$ R96 581,81 = R684,09
Amount at the end of the eleventh month	R96 581,81 + R684,09 = R97 265,90
Interest earned after the twelfth month	$\frac{0,7083}{100} \times$ R97 265,90 = R688,93
Amount at the end of the twelfth month	R97 265,90 + R688,93 = R97 954,83

(a) Mr Mahou earned R97 954,83 - R90 000 = R7 954,83 interest.
This is R7 954,83 – R7 650 = R 304,83 more than he would have earned had he taken out the interest.

(b) % interest:

$$\% = \frac{7\,954,83}{90\,000} \times 100$$

$$= 8,84\%$$

It is 8,84% – 8,5% = 0,34% more

5.6 Yes, by R7 954,83. The interest is added onto the capital each month so the interest is slightly more each month.

5.7

% interest earned per month	8,5% 12 = 0,7083%
Interest earned after the first month	$\frac{0,7083}{100} \times$ R90 000 = R637,50
Amount at the end of the first month	R90 000 + R200 = R90 200
Interest earned after the second month	$\frac{0,7083}{100} \times$ R90 200 = R638,89
Amount at the end of the second month	R90 200 + R200= R90 400
Interest earned after the third month	$\frac{0,7083}{100} \times$ R90 400 = R640,30
Amount at the end of the third month	R90 400 + R200= R90 600
Interest earned after the fourth month	$\frac{0,7083}{100} \times$ R90 400 = R641,72
Amount at the end of the fourth month	R90 600 + R200= R90 800
Interest earned after the fifth month	$\frac{0,7083}{100} \times$ R90 800 = R643,14
Amount at the end of the fifth month	R90 800 + R200= R91 000
Interest earned after the sixth month	$\frac{0,7083}{100} \times$ R91 000 = R644,55
Amount at the end of the sixth month	R91 000 + R200= R91 200
Interest earned after the seventh month	$\frac{0,7083}{100} \times$ R91 200 = R645,97
Amount at the end of the seventh month	R91 200 + R200= R91 400
Interest earned after the eighth month	$\frac{0,7083}{100} \times$ R91 400 = R647,38
Amount at the end of the eighth month	R91 400 + R200= R91 600
Interest earned after the ninth month	$\frac{0,7083}{100} \times$ R91 600 = R648,80
Amount at the end of the ninth month	R91 600 + R200= R91 800
Interest earned after the tenth month	$\frac{0,7083}{100} \times$ R91 800 = R650,22
Amount at the end of the tenth month	R91 800 + R200= R92 000
Interest earned after the eleventh month	$\frac{0,7083}{100} \times$ R92 000 = R651,64
Amount at the end of the eleventh month	R92 000 + R200= R92 200
Interest earned after the twelfth month	$\frac{0,7083}{100} \times$ R92 200 = R653,05
Amount at the end of the twelfth month	R92 200 + R200= R92 400

5.8 His investment grew by R2 400.

5.9 R653,05 – R637,50 = R15,55

5.10 He now receives only 8% per annum which means he will have less money to live on. If he was taking out all the interest, he would only receive R600 per month which is R37,50 less each month than he originally received. If he invests his money it will grow more slowly. For people who owe money to the bank, a decrease in the interest rate is a good thing, but for people like Mr Mahou it is not.

5.11 He should try to save as much as he can every month so that his capital will grow.

Activity 5 — All about interest

Mr Mahou has retired and receives a small pension. He does, however, have R90 000 which he decides to put into the bank . His pension is not enough to live off so he needs to use some of the interest he gets from his R90 000 every month. When Mr Mahou invested his money the interest rate was 8,5 % per annum compounded monthly.

Answer the following questions.

5.1 What is the interest rate per month if the bank determines this by dividing the annual rate by the number of months?

5.2 How much interest would Mr Mahou receive every month on the R90 000?

5.3 How much interest did he earn in total over the whole year?

5.4 Did his investment grow at all in the year? Explain your answer.

5.5 One day Mr Mahou decides not to use his interest every month but rather to leave it in the bank so that it is added to the account balance every month.

(a) Redraw the following table and use it to calculate how much interest he will earn. The first month has been done for you.

Original amount	R90 000
% interest earned per month	8,5% ÷ 12 = 0,7083%
Interest earned after the first month	$\frac{0,7083}{100} \times$ R90 000 = R637,50
Amount at the end of the first month	R90 000 + R637,50 = R90 637,50
Interest earned after the second month	
Amount at the end of the second month	
Interest earned after the third month	
Amount at the end of the third month	
Continue till the end of the year.	

How much interest did Mr Mahou earn in total in this way? How much more interest did he earn than when he took the interest out each month?

(b) Using the interest that you calculated in (a) and work out the actual percentage interest that Mr Mahou earned for the year. How much more was it than the quoted 8,5%?

5.6 Did his original investment grow at all? If so, by how much? Why is this more than the interest he earned in question 5.3?

5.7 After doing these calculations Mr Mahou decides that he needs the money to live on every month but also sees the need to save something every month. He decides to take the interest and put R200 back into his investment. Redraw the following table and calculate the growth in his investment after a year.

Interest earned after the first month	$\frac{0,7083}{100} \times R90\,000 = R637,50$
Amount at the end of the first month	R90 000 + R200 = R90 200
Interest earned after the second month	
Amount at the end of the second month	
Interest earned after the third month	
Amount at the end of the third month	
Continue till the end of the year.	

5.8 By how much did his investment grow?

5.9 How much more money is he getting at the end of the last month, compared to the first month that he invested his money?

5.10 Mr Mahou reads in the newspaper that the prime interest rate has decreased by 0,5%. Discuss how this will affect Mr Mahou's situation.

5.11 We have looked at Mr Mahou's investment for only 1 year, which is a fairly short time. What would you recommend that he does with his money?

Activity 6 — Buying a new fridge

ABOUT THIS ACTIVITY

The focus of this activity is on interest rates and on making good decisions when buying large items. The students will investigate 2 options, namely paying a company in instalments and borrowing money from the bank. This activity could be used as an assessment activity as the same calculations have been used in the previous activity. This activity addresses all the assessment criteria of SO2 of the personal finance unit standard.

MANAGING THIS ACTIVITY

This activity is completely self-contained. Students will be working with percentages and will need a calculator. An Excel spreadsheet can be used for repetitive calculations.

6.1 Amount paid = R158 × 24 months + R300 (deposit) = R4 092.

6.2 R4 092 – R2 999 = R1 093 more if paid in instalments.

6.3 Interest charged per month = $\frac{21}{12}$ = 1,75%

6.4

Price of fridge	R2 999
Deposit paid	R300
Amount to be paid off	R2 699
Interest owed after 1st month ∴ Amount owing after 1st month	$\frac{1,75}{100}$ × R2 699 = R47,23 R2 699 + R47,23 = R2 746,23
Amount owing after 1st payment	R2 746,23 – R400 = R2 346,23
Interest owed after 2nd month ∴ Amount owing after second month	$\frac{1,75}{100}$ × R2 346,23 = R41,06 R2 346,23 + R41,06 = R2 387,29
Amount owing after 2nd payment	R2 387,29 – R400 = R1 987,29
Interest owed after 3rd month ∴ Amount owing after 3rd month	$\frac{1,75}{100}$ × R1 987,29 = R34,78 R1 987,29 + R34,78 = R2 022,07
Amount owing after 3rd payment	R2 022,07 – R400 = R1 622,07
Interest owed after 4th month ∴ Amount owing after 4th month	$\frac{1,75}{100}$ × R1 622,07 = R28,39 R1 622,07 + R28,39 = R1 650,46
Amount owing after 4th payment	R1 650,46 – R400 = R1 250,46
Interest owed after 5th month ∴ Amount owing after 5th month	$\frac{1,75}{100}$ × R1 250,46 = R21,88 R1 250,46 + R21,88 = R1 272,34
Amount owing after 5th payment	R1 272,34 – R400 = R872,34
Interest owed after 6th month ∴ Amount owing after 6th month	$\frac{1,75}{100}$ × R872,34 = R15,27 R872,34 + R15,27 = R887,61
Amount owing after 6th payment	R887,61 – R400 = R487,61
Interest owed after 7th month ∴ Amount owing after 7th month	$\frac{1,75}{100}$ × R487,61 = R8,53 R487,61 + R8,53 = R496,14
Amount owing after 7th payment	R496,14 – R400 = R96,14
Interest owed after 8th month ∴ Amount owing after 8th month	$\frac{1,75}{100}$ × R96,14 = R1,68 R96,14 + R1,68 = R97,82
Amount owing after 9th payment	Nil
Number of months taken to pay off the fridge	8 months

6.5 R300 (deposit) + 7 (payments) × R400 + R97,82 (last payment) = R3 197,82

6.6 R4 092 – R3 197,82 = R894,18

6.7

Price of fridge	R2 999
Deposit paid	R300
Amount to be paid off	R2 699
Interest owed after 1st month ∴ Amount owing after 1st month	$\frac{1,75}{100}$ × R2 699 = R47,23 R2 699 + R47,23 = R2 746,23
Amount owing after 1st payment	R2 746,23 – R500 = R2 246,23
Interest owed after 2nd month ∴ Amount owing after second month	$\frac{1,75}{100}$ × R2 246,23 = R39,31 R2 246,23 + R39,31 = R2 285,54
Amount owing after 2nd payment	R2 285,54 – R500 = R1 785,54
Interest owed after 3rd month ∴ Amount owing after 3rd month	$\frac{1,75}{100}$ × R1 785,54 = R31,25 R1785,54 + R31,25 = R1 816,79
Amount owing after 3rd payment	R1 816,79 – R500 = R1 316,79
Interest owed after 4th month ∴ Amount owing after 4th month	$\frac{1,75}{100}$ × R1 316,79 = R23,04 R1 316,79 + R23,04 = R1 339,83
Amount owing after 4th payment	R1 339,83 – R500 = R839,83
Interest owed after 5th month ∴ Amount owing after 5th month	$\frac{1,75}{100}$ × R839,83 = R14,70 R839,83 + R14,70 = R854,53
Amount owing after 5th payment	R854,53 – R500 = R354,53
Interest owed after 6th month ∴ Amount owing after 6th month	$\frac{1,75}{100}$ × R354,53 = R6,20 R354,53 + R6.20 = R360,73
Amount owing after 6th payment	Nil
Number of months taken to pay off the fridge	6 months

Cost of fridge = R300 + 5 × R500 + R360,73 = R3 160,73
Money saved = R3 197,82 – R3 60,73 = R37,09

6.9 Bank overdraft:

Interest rate per month	$\frac{16,5}{12} = 1,375\%$
Amount to be paid off	R2 999
Interest owed after 1st month ∴ Amount owing after 1st month	$\frac{1,375}{100} \times$ R2 999 = R41,23 R2 999 + R41,23 = R3 040,23
Amount owing after 1st payment	R3 040,23 – R500 = R2 540,23
Interest owed after 2nd month ∴ Amount owing after second month	$\frac{1,375}{100} \times$ R2 540 = R34,93 R2 540,23 + R34,93 = R2 575,16
Amount owing after 2nd payment	R2 575,16 – R500 = R2 075,16
Interest owed after 3rd month ∴ Amount owing after 3rd month	$\frac{1,375}{100} \times$ R2 075,16 = R28,53 R2 075,16 + R28,535 = R2 103,69
Amount owing after 3rd payment	R2 103,69 – R500 = R1 603,69
Interest owed after 4th month ∴ Amount owing after 4th month	$\frac{1,375}{100} \times$ R1 603,69 = R22,05 R1 603,69 + R22,05 = R1 625,74
Amount owing after 4th payment	R1 625,74 – R500 = R1 125,74
Interest owed after 5th month ∴ Amount owing after 5th month	$\frac{1,375}{100} \times$ R1 125,74 = R15,48 R1 125,74 + R15,48 = R1 141,22
Amount owing after 5th payment	R1 125,74 – R500 = R641,22
Interest owed after 6th month ∴ Amount owing after 6th month	$\frac{1,375}{100} \times$ R641,22 = R8,82 R641,22 + R8,82 = R650,04
Amount owing after 6th payment	R650,04 - R500 = R150,04
Interest owed after 7th month ∴ Amount owing after 7th month	$\frac{1,375}{100} \times$ R150,04 = R2,06 R150,04 + R2,06 = R152,06
Number of months taken to pay off the fridge	7 months

6.9 Amount paid for fridge = 6 × R500 + R152,06 = R3 152,06

6.10 The students need to make a decision and motivate it from the data they have worked with. Some points to consider could be:

- It would be better to pay R158 over 24 months as it is a relatively small amount but would free up the monthly income for other expenses. The buyer would only be paying R894,18 more over 2 years.
- R400 per month would be the better option as it is cheaper than the 2 year option and yet it is not much more in total than paying R500 per month. This means that R100 per month is available for other expenses.
- The bank overdraft facility does not give much of an advantage.

Activity 6 — Buying a new fridge

You are need to buy a new fridge and you have been researching the best way to pay for it i.e. either cash or instalments. The advertisement says that on special you could purchase the size fridge you want for R2 999, if you pay cash. However you could also pay for the fridge by putting down a R300 deposit and then paying off R158 per month for 24 months (2 years). You would like to get the fridge as soon as possible but have not got enough money at the moment to pay cash. You need to investigate whether it would be more cost effective to buy it in instalments but reduce the number of months over which you would pay it off, or whether you should pay it off over the 24 month period.

6.1 Calculate what you would pay for the fridge if you paid over 24 months.

6.2 How much more would you pay for the fridge if you paid by instalments rather than cash?

6.3 The interest that is charged per annum (year) is 21%. However, the interest is added on every month so calculate the interest charged per month.

6.4 The shop that you are buying the fridge from agrees to let you pay more per month in order to reduce the number of months needed to pay off your loan. You decide that you can afford to pay R400 per month. Redraw the following table and calculate the number of months that you will take to pay off the fridge. The first month has been done for you.

Price of fridge	R
Deposit paid	R
Amount to be paid off	R
Interest owed after 1st month ∴ Amount owing after 1st month	$\frac{1,75}{100} \times$ R2 699 = R47,23 R2 699 + R47,23 = R2 746,23
Amount owing after 1st payment	R2 746,23 – R400 = R2 346,23
Interest owed after second month Amount owing after second month	
Amount owing after 2nd payment	
Repeat until you have paid off the fridge	
Number of months taken to pay off the fridge	

6.5 Calculate how much you actually paid for the fridge if you paid R400 per month.

6.6 How much did you save by paying R400 per month rather than R158 per month?

6.7 Repeat the calculation but pay off R500 per month. How much money do you save paying R500 rather than R400?

6.8 If you had overdraft facilities another option that you could consider is to take an overdraft with the bank to finance the fridge. The current overdraft rate is 16,5% per annum. In this case you are not paying a deposit but are borrowing the full amount of R2 999 so you would aim to pay the bank back R500 per month. Redraw the table and fill in the values.

Interest rate per month	
Interest owed after 1st month Amount owing after 1st month	
Amount owing after 1st payment	
Interest owed after second month Amount owing after second month	
Amount owing after 2nd payment	
Repeat until you have paid off overdraft	
Number of months taken to pay off overdraft Amount paid in the last month	

6.9 Calculate how much the fridge cost you using the bank's overdraft facility.

6.10 Now you need to make a decision.
The options are:

- Pay cash (but you can't afford this).
- Pay over 24 months with relatively low monthly repayments.
- Pay more per month and take fewer months to pay it off.
- Take an overdraft from the bank and pay the bank back (remember that you borrow the full amount from the bank).

Remember always to give reasons for your decision.

Activity 7 — Savings and investments.

This is the final activity of personal finance and looks at savings and investments.

ABOUT THIS ACTIVITY

This activity highlights the different types of saving options. It looks at the difference between short term and long term investments as well as low and high risk investments. This activity addresses AC1,AC2, AC3 and AC4 of the SO2 of the personal finance unit standard.

MANAGING THIS ACTIVITY

This activity consists of a worksheet and a handout. This activity could be expanded by getting the students to bring the personal finance section of the newspaper and compare interest rates of different banks as well as look at the unit trust performance. The students could also get pamphlets from the various banks and look at the variety of products available.

7.1 For retirement, for emergencies and unexpected expenses, to be able to purchase big items avoiding higher purchase interest, for vacations, for education etc.

7.2 Stokvels:
- (a) Amount received to spend = R150 × 12 = R1 800
- (b) No. By the time everyone has had a turn to get a lump sum you would have put in R1 800 and that is what you received when it was your turn.
- (c) You get a lump sum to use to purchase an item cash which means you can avoid hire purchase.
- (d) Your money doesn't grow and you have to trust that all the parties will pay their monthly contribution even after they have received their lump sum.

7.3 Current account:
- (a) R2 500
- (b) This account is used mainly for everyday transactions so the balance fluctuates all the time. This makes administration costs high. It also means that the bank can't invest your money to earn interest for them i.e. your money is not working for them so they can't pay you a high interest rate.
- (c) To encourage you to have large sums of money in their bank, to encourage you to save more.
- (d) The interest that is calculated is added onto your starting amount so if it is calculated daily it will grow daily so by the end of the year you will have earned more interest than if it was calculated at the end of the year. It also encourages you to save more money as it will earn interest as soon as you deposit it. You will effectively earn more than the quoted annual interest rate. (See activity 5)
- (e) 2,25%
- (f) R1 000 000 or more
- (g) No. Your money would be depreciating in value.

7.4 Savings account
- (a) R500
- (b) Saving accounts are used to save money in, not to do daily banking from, so the balance remains more constant so it is easier to administer.
- (c) 3,8%
- (d) R100 000

7.5 Fixed deposit:
- (a) You earn a higher interest once you have more than R10 000 in the bank.
- (b) You earn interest for any amount. Your money has to remain in this account for a definite time period which means the bank knows that your balance will be constant.
- (c) 5,25% You would only earn 0,5% if you put it into a savings account. Your money is accessible in a savings account.
- (d) 7,5%
- (e) 6,10%
- (f) Invest over R10 000 for more than 3 years.

7.6 Unit trusts:

(a) Profit = Net value price – price you bought the unit

(b) About R2,99

Total selling price = R2,99 × 500
= R1495

(c) Yes, because the selling price has dropped by about 42 cents

(d) About R1,08.

Total selling price = R1,08 × 1000
= R1080

(e) New price is R1,38

New total selling price = R1,38 × 1000
= R1380

Lost out on R1380 – R1080 = R300

(f) M Global income fund. You get the least return for your R100 as well as getting out less than you invested. The students could say it is a good time or bad time to sell! Good time because the value might drop further so you would lose even more. Bad time because the units might improve in value and then you would have lost money so leave them in longer.

(g) G fixed interest fund

(h) M global real estate. No.

(i) G fixed interest rate. Same as (g)

(j) M Global income fund and M global real estate. Yes because if you watch this funds performance carefully you could sell when it is doing well and earn good growth. No, because it does mean your could lose on your investment.

(k) 4,83%. No, best rate is 5,75% per annum which is 2,875% in 6 months.

Activity 7 — Savings and investments.

An important part of managing your personal finance is saving. As mentioned in activity 2, savings should be at least 10% - 15% of your monthly net income.

Some terminology:

LOW RISK investment

Has low interest rate — which means limited growth. However, you will get your money + interest, less bank charges.

HIGH RISK investment

Has the potential to earn large growth
BUT
you can also lose your money.

The MORE available your money is the LESS interest you earn. Unit trusts are an exception.

AVAILABILITY of your money

The LESS available your money is the GREATER the interest you earn.

7.1 Give at least three reasons why you should save.

7.2 Poorer communities often save as a group by means of a stokvel or umgalelo. Each person who is part of the scheme will put a fixed amount of money in each month. The total amount that is collected is given to one person in the scheme who can then use it. Each person in the scheme gets a month when they get a lump sum.

(a) If there are 12 members of a stockvel and they each put in R150 per month, how much will each member get to spend when it is his/her turn to receive the money.
(b) If you are part of a stokvel scheme, does your money grow (In other words, do you get out more money than you put in)? Explain your answer.
(c) What are the advantages of being in a stokvel?
(d) What are the risks or disadvantages of a stokvel scheme?

7.3 The handout provided shows you the interest rate for three types of bank accounts. Type 1 is a current account where your money is freely available. Your salary is usually deposited into this type of account and you can make payments, draw cheques and generally manage your money from this account.
(a) What balance do you have to have in this account before you start earning any interest on that money?
(b) Why do you think that the bank wants you to have this amount in your account before they will pay you interest?
(c) From the table you can see that the more money you have in the bank the greater the rate of interest. Why do you think that the banks do this and don't just give a fixed rate for any value?
(d) Interest rates are quoted per annum but the table says that interest is calculated on a daily basis and compounded monthly. Why is better to have your interest calculated on a daily basis and not just at the end of each year (Refer to activity 5 - all about interest)?
(e) What is the maximum interest you can earn in this account?
(f) How much do you have to have in this account to earn this maximum interest?
(g) If the inflation rate is 4%, would it be advisable to leave your money in this type of account? Discuss your answer.

7.4 Type 2 account is a savings account.
(a) What is the least amount of money you need to start earning interest?
(b) Why do you think that this account has a much lower starting point than the current account?
(c) What is the maximum interest that you can earn in this account?
(d) How much do you have to have in this account to earn this maximum interest?

7.5 Type 3 account is a fixed deposit account which requires you to leave your money in the bank for a certain amount of time.
(a) Why are there two rates given for each specified length of time?
(b) How much money do you have to have in this account before you can start earning any interest? This differs from the 2 previous accounts. Why?
(c) If you invest R1 200 in this account for 4 months, what interest rate will you be earning? Compare this to the interest that you would earn if you put it in a savings account (type 2). What would be the only advantage of putting it into a savings account ?
(d) If you invest R12 000 in this account for 5 years, what interest rate will you be earning?
(e) If you invest R2 500 in this account for 1½ years, what interest rate will you be earning?
(f) How could you get the best interest rate for this account?

TABLE 1 : INTEREST RATES OF TYPE 1 ACCOUNT — CURRENT ACCOUNT

Daily Balances	Interest rate
Below - R2 499	0,00%
R2 500 - R4 999	0,25%
R5 000 - R9 999	0,25%
R10 000 - R49 999	0,75%
R50 000 - R99 999	1,25%
R100 000 - R999 999	1,50%
R1 000 000 and above	2,25%
Interest rates are quoted on a per annum rate.	

TABLE 2 : INTEREST RATES OF TYPE 2 ACCOUNT — SAVINGS ACCOUNT

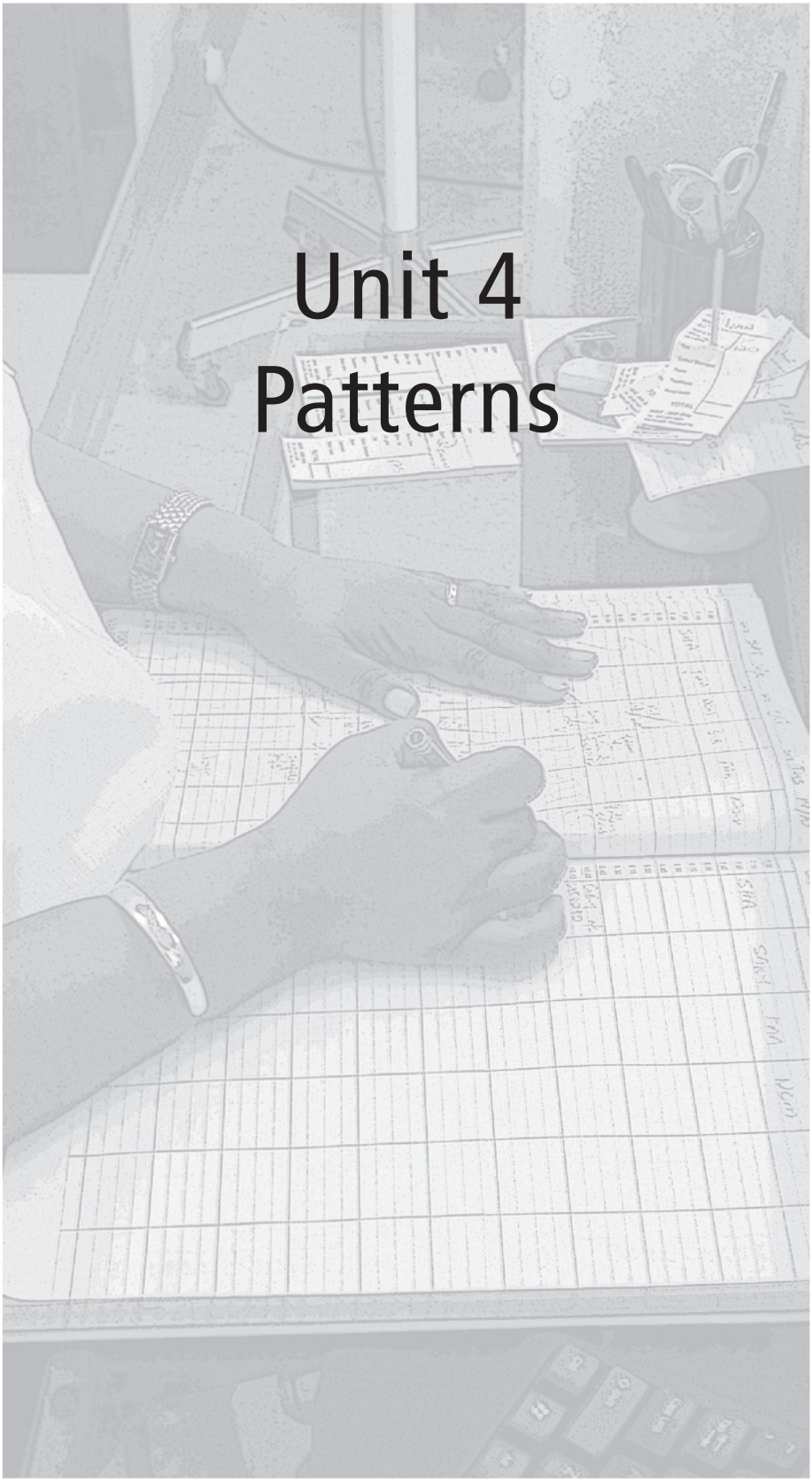
Daily Balances	Interest rate
Below - R499	0,00%
R500 - R999	0,25%
R1 000 - R5 999	0,50%
R6 000 - R9 999	1,00%
R10 000 - R19 999	1,50%
R20 000 - R49 999	2,00%
R50 000 - R74 999	2,25%
R75 000 - R99 999	2,50%
R100 000 and above	3,80%
Interest rates are quoted on a per annum rate.	

TABLE 3 : INTEREST RATES OF TYPE 3 ACCOUNT — FIXED DEPOSIT ACCOUNT

Length of time of investment	Balances below R10 000	Balances of R10 000 to R99 999
33 days to under 3 months	4,5%	5,50%
3 months to under 6 months	5,25%	6,25%
6 months to under 12 months	5,75%	6,75%
12 months to under 18 months	6,10%	7,10%
18 months to under 24 months	6,20%	7,20%
24 months to under 36 months	6,40%	7,40%
36 months to under 48 months	6,50%	7,50%
48 months to 60 months	6,50%	7,50%
R100 000 and above		On application.

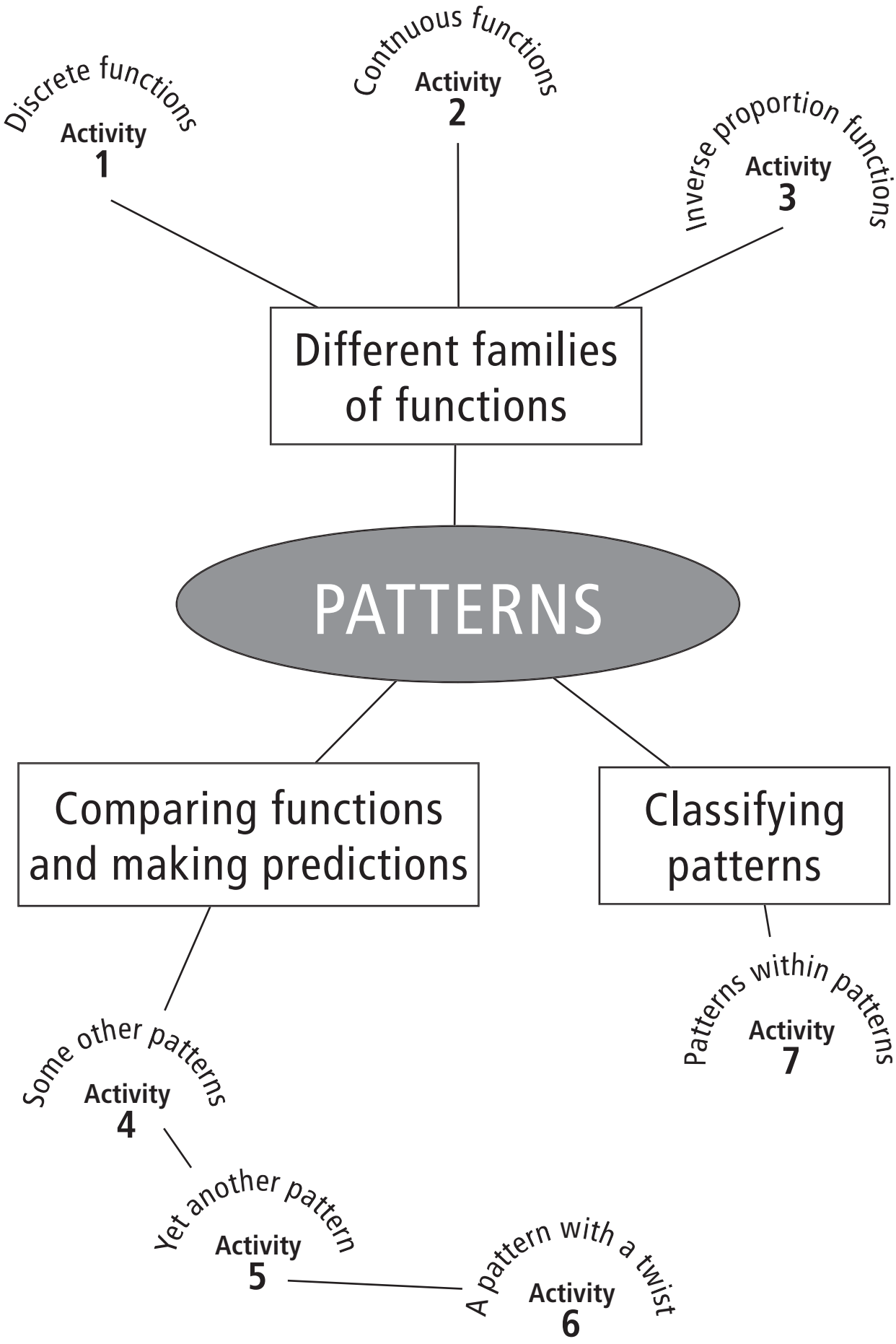
Mathematical Literacy

NQF LEVEL 2



Unit 4
Patterns

Generic Business Administration


Discrete functions
Activity 1
Contnuous functions
Activity 2
Inverse proportion functions
Activity 3
Different families of functions
PATTERNS
Comparing functions and making predictions
Classifying patterns
Some other patterns
Activity 4
Yet another pattern
Activity 5
A pattern with a twist
Activity 6
Patterns within patterns
Activity 7

Overview

This unit aims to develop a variety of mathematical skills including:

- Translating from one representation to another (i.e. verbal, tables, formulae, graphs).
- Dealing with situations involving the range of functions specified in the main range statement as well as functions for which there is no rule.
- Working with functions: identifying them, contrasting them and comparing the features of the functions listed in the main range statement.
- Working with functions for which there are no rules.
- Recognising equivalent forms of expressions, equations or functions.
- Reading and interpreting information represented in tables and graphs and extracting appropriate information from these.
- Using expressions, functions and equations to represent situations.
- Developing strategies for deciding whether symbolic representations are reasonable, and interpreting such results.

This unit consists of seven activities each of which can stand alone. They rely, however, on an understanding of the concepts that have been raised in preceding activities. Facilitators should consult the information given in activity seven before embarking on taking students through this unit in order to be clear about the underlying lesson each activity is intended to teach.

Having worked through this unit, students should have a greater ability to work with patterns and functions and represent them graphically. In developing the mathematical literacy materials the author team has, in general, tried hard to use authentic contexts to give meaning to the mathematics being studied. In this unit, however, the nature of the specific outcomes and assessment criteria shift the focus to the mathematical knowledge, skills, attitudes and values as opposed to contextual issues. The user will therefore find that while contexts are still used, they are, in general, less authentic.

With the discrete functions used in these activities we draw a graph by plotting all the points and making the observation that there are no points in between. This is not, however, an efficient way to draw these graphs. Students could draw a smooth curve through certain selected points, but must remember that this line can only be used for values which are possible within the context.

NOTE: In order to draw the graphs required in the activities students will need graph paper.

Unit outcomes

The following Unit Standards, Specific Outcomes and Assessment Criteria are addressed by this unit:

Work with a range of patterns and functions and solve problems (9007)

- Convert flexibly between and within various representations of functions (SO1).
  - o Appropriate information is selected to convert flexibly between and within various representations of functions.
  - o Appropriate representations are selected for specific applications.
  - o Conversions represent the functions accurately and appropriately.
- Compare, analyse and describe the behaviour of patterns and functions (SO2).
  - o Patterns and functions are compared in terms of:
    - i. Shape and symmetry.
    - ii. Finding function values.
    - iii. Finding input values.
    - iv. The average rate of change of function values.
  - o The key features of the graphs of functions are described and interpreted correctly.
  - o The behaviour of functions is described as being increasing or decreasing or constant as determined visually from graphical representations.
- Represent situations mathematically in order to interpret and solve problems (SO3).
  - o Accurate point-by-point plotting is used to model contextual problems.
  - o Appropriate symbolic representations are used to model contextual problems.
  - o Representations are analysed and manipulated efficiently in arriving at results.
  - o Representations are verified in terms of available data.
  - o Results are interpreted correctly in terms of the context.
  - o Interpretations and predictions are based on the properties of the mathematical model.

Activity 1: Discrete functions
In this activity we introduce students to discrete functions, namely functions for which the independent variable (input value) can only be a whole number. In particular, we work with the basic functions $y = ax$ and $y = ax + b$ where x is an element of the positive whole numbers.

Activity 2: Continuous functions
In this activity we study patterns which are different from those of activity 1. Students will explore continuous functions and try to identify the ways in which these patterns are both different from and similar to those we studied in activity 1.

Activity 3: Inverse proportion functions
In this activity students are introduced to another basic function, namely xy = k (a rectangular hyperbola). This function is useful in situations concerned with describing inverse proportion. The first situation deals with discrete data while the second deals with continuous data. It is easier to think of applications of the discrete function than of the continuous one. For this reason the second part of the activity deals with $xy = 64$ in the absence of a context.

Activity 4: Some other patterns
In this activity the underlying concept that is addressed is the notion that although there are numbers, there is not by implication a mathematical relationship (rule/function) that relates the numbers to each other. In the first and third of the situations studied in the activity, there is some discernable pattern. Our social knowledge of the situations can give us confidence to make predictions (to interpolate and extrapolate). However, there are no rules/functions that describe the situations. In the second of the situations there isn't even the possibility of using social knowledge of the context in order to make predictions, since the data are completely unrelated.

Activity 5: Yet another pattern
In this activity we look at functions which behave differently from those studied in the previous activities in that they are not smooth curves. Students are exposed to interesting situations where we look at functions which are a combination of a constant and a step function (i.e. within each of the clearly visible sections/intervals of the function, it remains constant).

Activity 6: A pattern with a twist
In this activity we work with functions of the form $y = ax + b$, where $a < 0$ (both discrete and continuous). Students work with tables and graphs of these functions and are encouraged to compare them with the other functions studied thus far in the unit.

Activity 7: Patterns within patterns
Over the first six activities we have explored a large number of different situations and ways of describing them. We have seen how the same situation can be represented by means of words, tables, graphs and formulae and we have translated from one representation to another. In this activity we will formally introduce vocabulary to describe what we have explored in the absence of such vocabulary.

9007	Work with a range of patterns and functions and solve problems									
		ACTIVITY	1	2	3	4	5	6	7
SO1	Convert flexibly between and within various representations of functions.								
	AC1	Appropriate information is selected to convert flexibly between and within various representations of functions.	✓	✓	✓	✓	✓	✓	✓
	AC2	Appropriate representations are selected for specific applications.	✓	✓	✓	✓	✓	✓	
	AC3	Conversions represent the functions accurately and appropriately.	✓	✓	✓		✓	✓	
SO2	Compare, analyse and describe the behaviour of patterns and functions.								
	AC1	Patterns and functions are compared in terms of: i. Shape and symmetry. ii. Finding function values. iii. Finding input values,. iv. The average rate of change of function values.	✓	✓	✓	✓	✓	✓	✓
	AC2	The key features of the graphs of functions are described and interpreted correctly.	✓	✓	✓	✓	✓	✓	✓
	AC3	The behaviour of functions is described as being increasing or decreasing or constant as determined visually from graphical representations.	✓	✓	✓		✓	✓	✓
SO3	Represent situations mathematically in order to interpret and solve problems.								
	AC1	Accurate point-by-point plotting is used to model contextual problems.	✓	✓	✓		✓	✓	
	AC2	Appropriate symbolic representations are used to model contextual problems.	✓	✓	✓		✓	✓	✓
	AC3	Representations are analysed and manipulated efficiently in arriving at results.	✓	✓	✓	✓	✓	✓	✓
	AC4	Representations are verified in terms of available data.	✓	✓	✓		✓	✓	
	AC5	Results are interpreted correctly in terms of the situation.	✓	✓	✓	✓	✓	✓	
	AC6	Interpretations and predictions are based on the properties of the mathematical model.	✓	✓	✓	✓	✓	✓	✓

Activity 1 — Discrete functions

ABOUT THIS ACTIVITY

In this activity we introduce students to discrete functions, namely functions for which the independent variable (input value) can only be a whole number. In particular, we work with the basic functions $y = ax$ and $y = ax + b$ where x is an element of the positive whole numbers.

This activity is aligned with unit standard 9007 and addresses AC 1, 2 and 3 of SO1; AC 1, 2 and 3 of SO2; and AC1, 2, 3, 4, 5, and 6 of SO3.

MANAGING THIS ACTIVITY

Students should be given sufficient time to explore and recognise the features of these functions and, in particular, what distinguishes the functions from each other. (In later activities students will look at what distinguishes discrete functions from continuous functions).

1.1 Completed answers in bold.

Number of conference satchels	1	2	3	4	5	6	7	8	9	10
Total cost (in rands)	24	48	72	**96**	**120**	**144**	**168**	**192**	**216**	**240**

1.2 The following responses can be expected:

- 3 × 24 = 72 ; 4 × 24 = 96 etc … (the functional relationship where we relate input to output).

Number of conference satchels	1	2	3	4	5	6	7	8	9	10
Total cost (in rands)	24	48	72	96	120	144	168	192	216	240

×24 ×24 ×24

- 24 + 24 = 48 ; 48 + 24 = 72 etc … (recognition of the recursive nature of the relationship).

Number of conference satchels	1	2	3	4	5	6	7	8	9	10
Total cost (in rands)	24	48	72	96	120	144	168	192	216	240

+24 +24

- Since 2 × 2 = 4 and 48 × 2 = 96 it follows that 4 satchels cost R96,00.

Number of conference satchels	1	2	3	4	5	6	7	8	9	10
Total cost (in rands)	24	48	72	96	120	144	168	192	216	240

×2

1.3 Gina could use the last column in the table. Since 10 × 2 = 20, the total cost for 20 satchels must be 240 × 2 = R480,00. Alternatively, she could use the functional relationship by doing the calculation: 24 × 20 = 480,00.

1.4.1 She can buy 25.

1.4.2 Although the ideal is for the students to work backwards and use a "rule" or formula (600 ÷ 24 = 25), some students may simply use a trial and improvement method to arrive at the answer of 25.

1.4.3 There are two options:

- She could say: since 120 × 5 = 600 and 5 × 5 = 25, she can buy 25 satchels.

Number of conference satchels	5	× 5	25
Total cost (in rands)	120		600

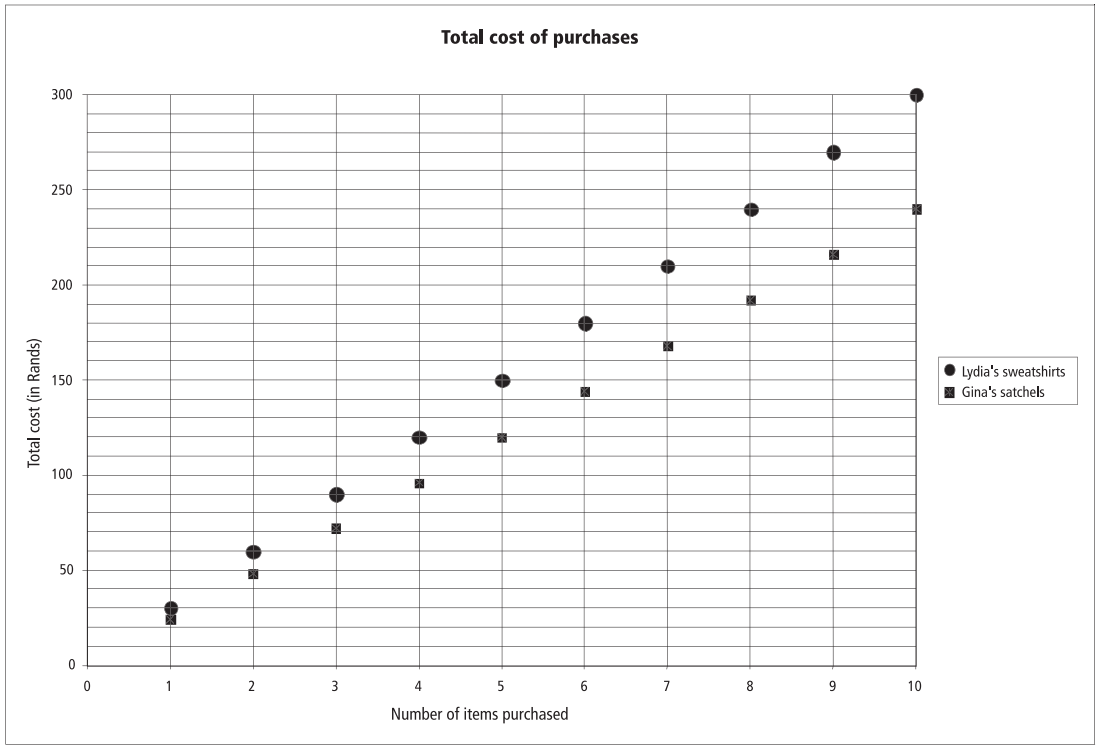
- Or she could approach the problem as follows:

  "For R240,00 she will get 10 satchels and still have R600,00 – R240,00 = R360,00 over.

Another 10 satchels will cost another R240,00 and there would be R360,00 – R240,00 = R120,00 over.
For the remaining R120,00 she can get 5 satchels.
In total she can get 10 + 10 + 5 = 25 satchels."

1.5 No, it would not make sense to join the points because the data is not continuous. You cannot buy parts of a sweatshirt, you can only buy whole sweatshirts. The key idea for students to grasp here is that with this discrete function the input number must be a whole number.

1.6



1.7.1 A sweatshirt is more expensive than a satchel.

1.7.2 You can see that for say two sweatshirts the point lies above the point for two satchels. You can also see that the slope (gradient) of the sweatshirt points is steeper than the slope of the satchel points.

1.7.3 The price of one sweatshirt is R30,00. This can be read off the graph.

1.8 Completed answers in bold

Number of tables	1	2	3	4	5	6	7	8		**12**
Number of chairs	4	6	8	**10**	**12**	14	**16**	**18**		24

1.9 The following responses can be expected:

- 4 + 2 = 6; 6 + 2 = 8 etc…(recognition of the recursive nature of the relationship)

Number of tables	1	2	3	4	5	6	7	8		**12**
Number of chairs	4	+2 6	+2 8	**10**	**12**	14	**16**	**18**		24

- A trial and improvement method or some other method which can be explained.
- It is unlikely that the more formal rule/function ("multiply the number of tables by 2 and then add 2" or simply "2n+2") will be obtained at this stage.

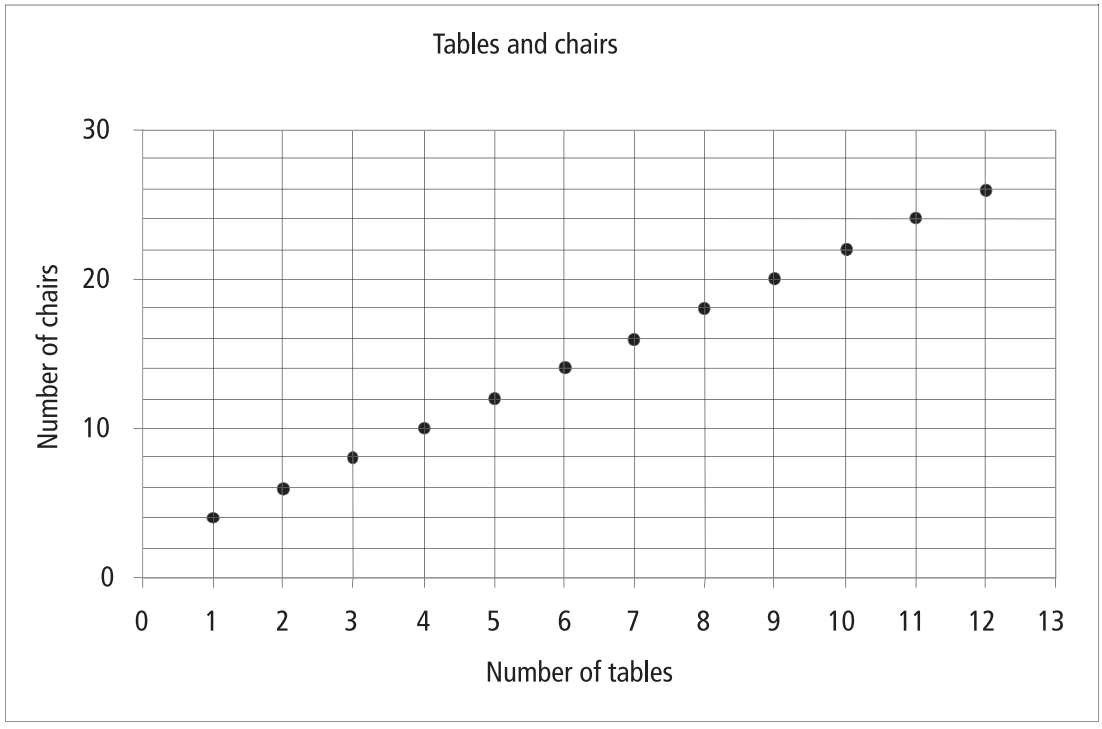
1.10 Students could add three lots of 2 to the answer at the end of the table (using the recursive relationship) to get 24 + 2 + 2 + 2 = 30.
With time we hope that they will try to find a functional relationship.

Unlike the example of the satchels, however, students should realise that they cannot simply say "since 5 × 3 = 15, it follows that 12 × 3 = 36 chairs will be needed." This method works for the satchels—a direct proportion function—but not for the tables and chairs—a linear function.

1.11.1 For 32 people (chairs) we would need 15 tables. While the more formal calculation could look as follows: (32 – 2) ÷ 2 = 15, students may, at this stage, prefer to use a trial and improvement approach.

1.11.2 Whichever method students use to do the calculations they should hopefully realise that 16 tables is not enough (16 × 2 + 2 = 34) and 17 tables gives too many chairs (17 × 2 + 2 = 36). Given the context you cannot have half tables and so they should choose 17 tables otherwise one of the 35 people would be without a seat.

1.12



1.13 No, it would not make sense to join the points because the data is not continuous. You cannot use parts of a chair or table, you can only use whole chairs or whole tables. The key idea for learners to grasp here is that with discrete functions the input number must be a whole number.

1.14 SAME: All of the graphs are increasing (going up) and the slope of each graph (the rate of change) corresponds to the constant difference between consecutive terms in the table. For example, as the number of satchels increases by one, so the price increases by R24,00 – this is reflected in the points of the graph which go up twenty four units for every one unit across.

DIFFERENT: For all the graphs, excluding the tables and chairs graph, students should notice that the value on the vertical axis is found by multiplying the value on the horizontal axis by the rate of change. However, for the tables and chairs graph the value on the vertical axis is found by multiplying the value on the horizontal axis by the rate of change and then adding a constant value (in this case, the constant is 2). You may also want to show the students that for each of the graphs except the table and chairs graph, a line drawn through the points would go through the origin (the point where the horizontal and vertical axes meet). In the case of the tables and chairs graph, the line joining the points does not go through the origin but cuts the vertical axis at the constant value.

1.15.1 In line with the assessment criteria, students should be encouraged to give both verbal and algebraic forms of the formulae and be able to recognise their equivalence.

In **words**, the formula can be given by: Cost = the number of conference satchels multiplied by the cost per conference satchel.

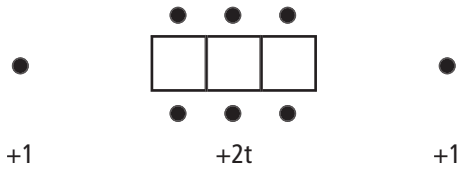
Algebraically, we could write: C= n × 24, where:
- o C is the total cost (in rands),
- o n is the number of conference satchels purchased, and
- o 24 is the price per conference satchel (in rands).

Similarly, the other formulae can be given in words or algebraically. However only the algebraic formulae have been listed below.

1.15.2 For sweatshirts: C = n × 30, where:
- o C is the total cost (in rands),
- o n is the number of sweatshirts purchased, and
- o 30 is the price per sweatshirt (in rands).

1.15.3 For tables and chairs: C = 2t + 2, where:
- o C is the number of chairs,
- o + 2 represents the chairs at the head and foot of the tables.
- o 2t represents the two chairs at each table (ignoring those at the head and foot).

Diagrammatically we could think of this as follows:



1.16 There are two kinds of formulae: one of the form $y = ax$ and the other of the form $y = ax + b$. The conference satchel and sweatshirt formulae are all the $y = ax$ form, while the tables and chairs formula is the $y = ax + b$ form. In time we expect students to associate "a" with the rate of change of the functions (gradient/slope of the graph) and "b" with the constant value (the vertical axis intercept).

Activity 1 — Discrete functions

This is the first of several activities in which we will be studying patterns—sometimes referred to as functions by mathematicians. We will clarify vocabulary such as "discrete functions"—the title of this activity—in a later activity. For now we will focus our study on different situations and the number patterns that arise from them.

Conference satchels

Gina is the receptionist for a company that sells conference satchels. Because the sales manager is very busy, he has started making a table for Gina that she can use to tell people who enquire after the cost of satchels for their conference how much they will pay for a given number of satchels.

Number of conference satchels	1	2	3	4	5	6	7	8	9	10
Total cost (in rands)	24	48	72							

1.1 Copy and complete the table.

1.2 Describe, in words, how you calculated the total cost in order to complete the table. Can you think of another way that you could have done this?

1.3 Discuss at least two different ways in which Gina could use this table to calculate the price for 20 satchels.

1.4 A client phones and tells Gina that his company has R600,00 with which to buy satchels. He asks Gina how many satchels they can buy with this amount of money.

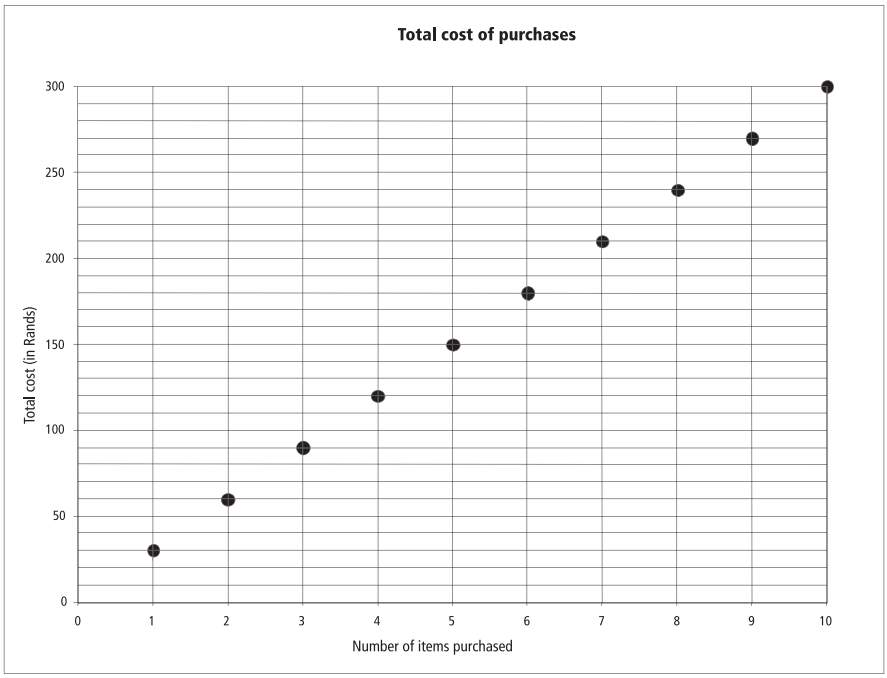
1.4.1 How many can they buy?

1.4.2 Describe how you calculated this.

1.4.3 How could Gina use the values in her table to determine how many satchels they can buy?

Lydia's sweatshirts

Lydia works for a company that sells sweatshirts for conference delegates. Instead of a table, she has decided to use a graph for finding the total cost. Her graph is drawn below:



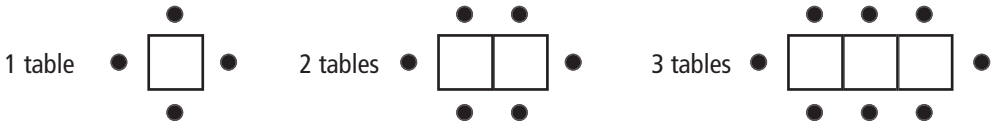
1.5 Notice that Lydia has not joined the dots on her graph. Would it make sense to do so? Give a reason for your answer.

1.6 Draw a graph of Gina's satchels on the same set of axes.

1.7 Compare the two graphs:
1.7.1 Which item is more expensive – a satchel or a sweatshirt?
1.7.2 How can you see from the graph which item is more expensive?
1.7.3 What is the price of one sweatshirt?

Tables and chairs

Frank sets up the tables for the dinner functions at a hotel. The pictures below show how the tables and chairs are arranged (the tables are always arranged in a straight line):



Frank has made the table below on which he has recorded the number of tables and chairs needed for different functions.

Number of tables	1	2	3	4	5	6	7	8		
Number of chairs	4	6	8			14				24

1.8 Copy and complete the table.

1.9 Describe, in words, how you determined the number of chairs in order to complete the table. Can you think of another way that you could have done this?

1.10 Discuss at least two different ways in which Frank could use this table to calculate the number of chairs needed if he uses 15 tables.

1.11 How many tables would he need if there were:
1.11.1 32 people attending a function?
1.11.2 35 people attending a function?

1.12 Draw a graph of the information in your completed table.

1.13 Have you joined the dots on your graph? Would it make sense to do so? Give a reason for your answer.

1.14 In what ways are the graphs for the satchels, sweatshirts and the tables and chairs the same? In what ways are they different?

1.15 Write a formula for calculating:
1.15.1 The total cost of buying different numbers of satchels.
1.15.2 The total cost of buying different numbers of sweatshirts.
1.15.3 The total number of chairs needed for different numbers of tables.

1.16 In what ways are the formulae the same? In what ways are they different?

Activity 2 — Continuous functions

ABOUT THIS ACTIVITY

In this activity we study patterns which are different from those of activity 1. Students will explore continuous functions and try to identify the ways in which these patterns are both different from and similar to those we studied in activity 1.

This activity is aligned with unit standard 9007 and addresses AC 1, 2 and 3 of SO1; AC 1, 2 and 3 of SO2; and AC1, 2, 3, 4, 5, and 6 of SO3.

MANAGING THIS ACTIVITY

Students should be given sufficient time to explore and recognise the features of these functions and, in particular, what distinguishes the functions from each other and from the discrete functions studied in activity 1.

2.1 Completed answers in bold.

Distance travelled (in kilometres)	15	20	25	30		52		108		144
Travelling allowance paid (in rands)	22,50	30,00	37,50	45,00		78,00		162,00		216,00

2.2 As with question 1.2 of activity 1, it is anticipated that the students will use one of the following three methods:

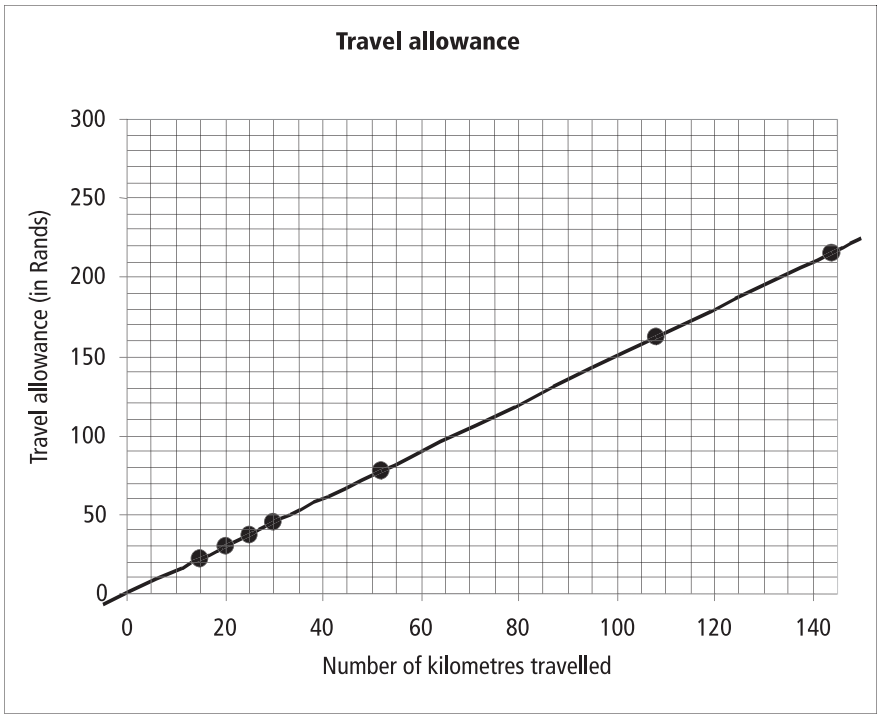
- 15 × 1,50 = 22,50; 20 × 1,50 = 30,00 etc…(the functional relationship where we relate input value to output value).
- 22,50 + 7,50 = 30,00; 30,00 + 7,50 = 37,50 etc.… (recognition of the recursive nature of the relationship).
- Since 15 × 2 = 30 and 22,50 × 2 = 45,00 it follows that 30km costs R30,00.

2.3 One could use the 1st number column in the table. Since 15 × 3 = 45 the total cost for 45km must be 22,50 × 3 = 67,50. Alternatively, one could use the functional relationship, by doing the calculation: 45 × 1,50 = R67,50.

2.4.1 The worker travelled a total of 81km.

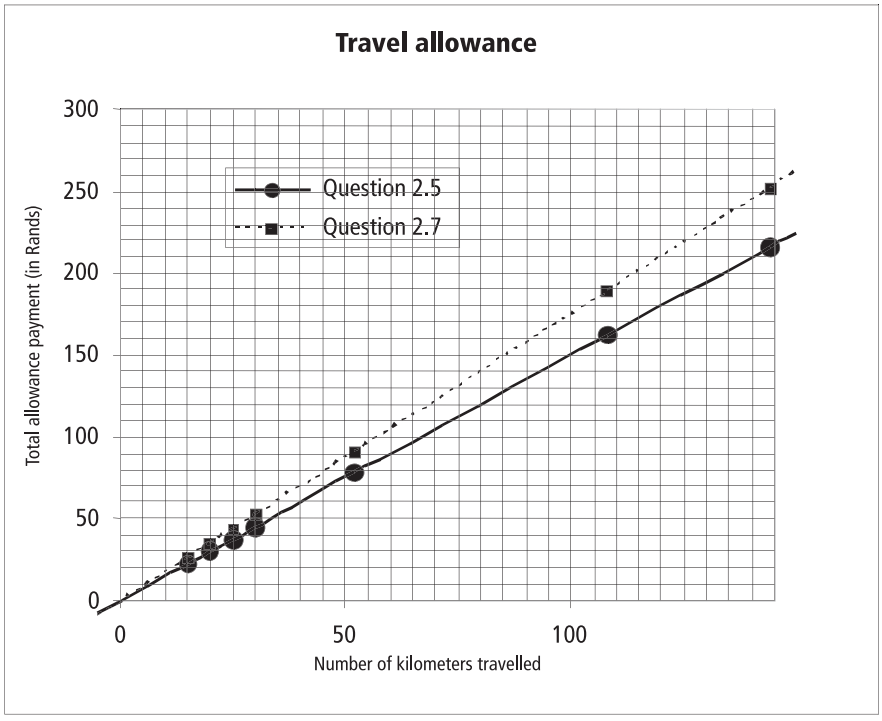
2.4.2 Since you can calculate the total allowance payment by multiplying the number of kilometers by 1,50, it follows that if we know how much the total allowance payment is we can divide this by 1,50 to get the number of kilometers travelled. (121,50 ÷ 1,50 = 81km).

2.5



2.6 Yes, the points on the graph have been joined. In this case, unlike the graphs in activity 1, the data is continuous. In other words, you can pay for parts of a kilometre. To develop the graph we used the values from the table. Because the data is continuous we can draw a line through the points we know and use it to determine costs for other distances *including parts of kilometres*. (NOTE: We could have used a similar line with the discrete function problems but in that case the line would not apply for values other than whole number x-values—and in those activities we deliberately discouraged students from drawing the line). The key idea for students to grasp here is that with continuous functions the input number does not need to be a whole number.

2.7



2.8.1 The new allowance graph lies above the old one and therefore you know that for each x-value (number of kilometres travelled) the y-value (total allowance in rands) is greater. Also, because the new allowance graph is steeper than the previous one (i.e. the gradient has increased) you know that the rate of change (refer to activity one) has increased.

2.8.2 SAME: Both graphs start at the origin (unlike the graphs in activity one) and both of the graphs are increasing (going up).

DIFFERENT: The slope of each graph (the rate of change) is different and corresponds to the constant difference between terms in each of the tables.

2.9 Completed values in bold.

Duration of the conference call (in minutes)	10	20	25	30	35		42		**68**
Total cost of the call (in rands)	100	150	175	**200**	**225**		260		390

2.10 The following responses can be expected:

- The recursive method (refer to activity one): Over a period of 10 minutes the cost increases by R50,00 and for a period of 5 minutes the cost increases by R25. Therefore the cost (rate) is R50 for every 10 minutes which is the same as R25 for every 5 minutes (which is the same as R5 per minute).

Duration of the conference call (in minutes)	10	+10	20	+5	25	30	35		42		**68**
Total cost of the call (in rands)	100	+50	150	+25	175	**200**	**225**		260		390

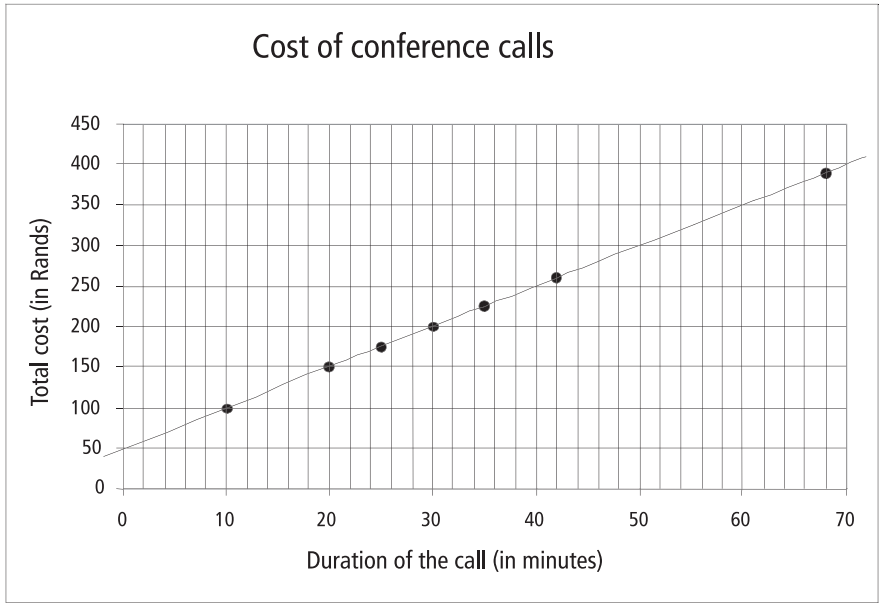
- A trial and improvement or some other method which can be explained.
- It is unlikely that the more formal rule/function ("multiply the number of minutes by 5 and add 50 or simply "5n+50") will be obtained at this stage.

2.11 Since the cost is R50 for 10 minutes this means that the cost per minute/rate is R5,00. The setup fee can be calculated by subtracting this cost (number of minutes × 5) from the total cost: therefore the setup fee is R50,00.

2.12 Since the total cost = 50 + 5 × number of minutes, the price for a 2hr (120 minute call) would be calculated as follows: 50 + 5 × 120 = R650,00.

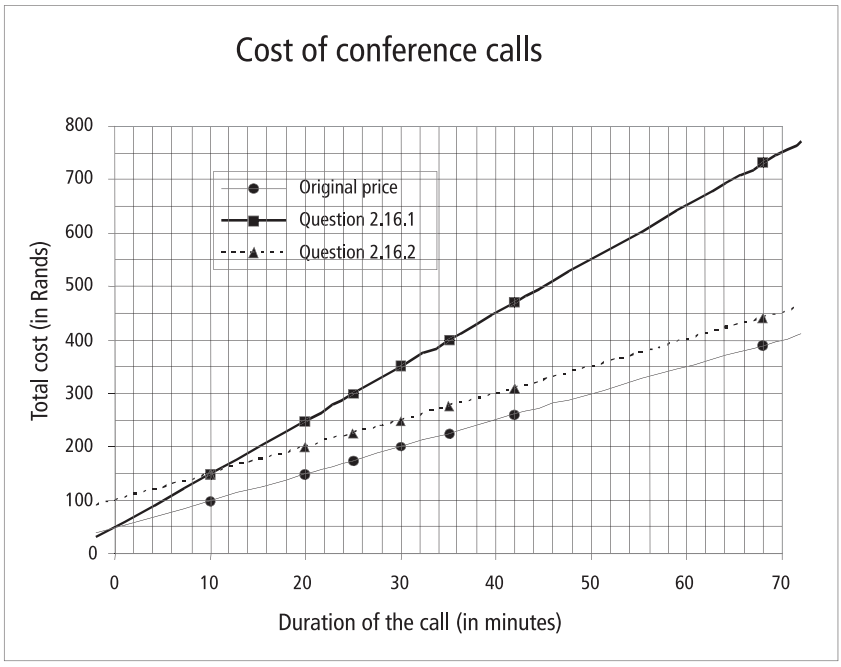
2.13 If we subtract the setup fee of R50,00 from R475,00 we get R425,00. Since the cost per minute (rate) is R5,00, we simply divide R425,00 by 5 and get 85 minutes. The duration of the call was 85 minutes.

2.14



2.15 Yes, the points on the graph have been joined. In this instance the data is again continuous (see earlier discussion).

2.16



2.17 SAME: All of the graphs are increasing (going up). The slope (the rate of change) of the original graph and the graph from question 2.16.2 are the same. Both the original graph and the graph from question 2.16.1 begin on the y-axis at 50 because the setup fee is R50,00.

DIFFERENT: The slope (rate of change) of the graph from question 2.16.1 is steeper because the cost per minute has increased. The graph of 2.16.2 begins on the y-axis at R100 because the setup fee has been doubled (R50 × 2).

2.18.1 A = 1,50d where:
- A is the total allowance (in rands),
- d is the total distance travelled, and
- "1,50" is the rate per kilometer (in rands).

2.18.2 A = 1,75d where:
- A is the total allowance (in rands),
- d is the total distance travelled, and
- "1,75" is the rate per kilometer (in rands).

2.18.3 C= 5n+50 where:.
- C is the total cost (in rands),
- n is the duration of the call (in minutes),
- "5" is the cost (in rands) per minute, and
- "50" is the setup fee of R50,00.

2.18.4 C= 10n+50 where:
- C is the total cost (in rands),
- n is the duration of the call (in minutes),
- "10" is the cost (in rands) per minute, and
- "50" is the setup fee of R50,00.

2.18.5 C= 5n+100 where:
- C is the total cost (in rands),
- n is the duration of the call (in minutes),
- "5" is the cost (in rands) per minute, and
- "100" is the setup fee of R100,00

2.19 There are two kinds of formulae: one of the form $y = ax$ and the other of the form $y = ax + b$. The two travel allowance formulae are the $y = ax$ form, while the conference call formulae are the $y = ax + b$ form. Students should by now have started to associate "a" with the rate of change of the functions (gradient/ slope of the graph) and "b" with the constant value (the vertical axis intercept).

Activity 2 — Continuous functions

This is the second of several activities in which we continue our study of functions. As mentioned earlier we will delay our discussion on vocabulary till later, focusing our study (for now) on different situations and the number patterns that arise from them. The patterns we study in this activity are different to those of activity 1. It would be a good idea for you to try to identify the ways in which the patterns in this activity are both different from and similar to those we studied in activity 1.

Petrol consumption

The charitable organisation for which you work provides support to terminally-ill patients, many of whom are bed-ridden. The organisation has decided to pay its field workers a travelling allowance for when they visit patients at their homes. The table below shows the amounts that have been paid to several field workers over the last few days

Distance travelled (in kilometres)	15	20	25	30		52		108		
Travelling allowance paid (in rands)	22,50	30,00	37,50	45,00						216,00

2.1 Copy and complete the table.

2.2 Describe, in words, how you calculated the travelling allowance in order to complete the table. Can you think of another way that you could have done this?

2.3 Discuss at least two different ways in which you could use this table to calculate the allowance for a 45 kilometer journey.

2.4 The travelling allowance "pay slip" for one of the workers reads R121,50.
2.4.1 How far did she travel?
2.4.2 Describe how you calculated this.

2.5 After using the table for some time the accountant has decided that the table is not that useful and decides to rather use a graph. Help the accountant by drawing the graph for her.

2.6 Look at the graph that you have drawn. Did you join the points on your graph? Give a reason for your answer.

2.7 The organisation decides to increase the travelling allowance by 25c per kilometer. Draw a graph for the new allowance onto the same set of axes that you used in 2.5.

2.8 Compare the two graphs:
2.8.1 How can you see from the graphs that the new allowance is more?
2.8.2 How are the two graphs the same and how are they different?

Conference calls

One service that is often needed during a conference is a conference call, to allow people in a different venue or country to participate in the discussions without travelling to the venue. The cost for this service is made up of two components: a setup fee and the cost of the connection which is based on the duration of the call. You have recorded the amounts that you were charged for several conference calls over the last year.

Duration of the conference call (in minutes)	10	20	25	30	35		42		
Total cost of the call (in rands)	100	150	175				260		390

2.9 Copy and complete the table.

2.10 Describe, in words, how you determined missing information in the table. Can you think of another way that you could have done this?

2.11 What is the setup fee for a conference call? What is the charge per minute for the connection? Describe how you determined these.

2.12 How much would a two-hour conference call cost?

2.13 If a particular conference call cost R475,00 how long was the call?

2.14 Draw a graph of the information in your completed table.

2.15 Have you joined the dots on your graph? Would it make sense to do so? Give a reason for your answer.

2.16 On the same set of axes that you used in 2.14 draw two more graphs as follows:

2.16.1 Let one graph represent the cost of the conference call if the setup fee is unchanged but the per-minute rate has doubled compared to the original graph.

2.16.2 Let the other graph represent the cost of the conference call if the setup fee has doubled but the per-minute rate is unchanged compared to the original graph.

2.17 In what ways are the different graphs for the traveling allowance and conference calls the same? In what ways are they different?

2.18 Write a formula for calculating:

2.18.1 The total travelling allowance for different distances travelled at the original rate.

2.18.2 The total travelling allowance for different distances travelled at the increased rate.

2.18.3 The total cost of a conference call in terms of the duration of the call (based on the original table).

2.18.4 The total cost of a conference call in terms of the duration of the call based on the original setup fee but increased per-minute rate.

2.18.5 The total cost of a conference call in terms of the duration of the call based on the increased setup fee and original per-minute rate.

2.19 In what ways are the formulae the same? In what ways are they different?

Activity 3 — Inverse proportion

ABOUT THIS ACTIVITY

In this activity students are introduced to another basic function, namely xy = k (a rectangular hyperbola). This function is useful in situations for describing inverse proportion. The first situation deals with discrete data while the second deals with continuous data. It is easier to think of applications of the discrete function than of the continuous one. For this reason the second part of the activity deals with xy = 64 in the absence of a context.

This activity is aligned with unit standard 9007 and addresses AC 1, 2 and 3 of SO1; AC 1, 2 and 3 of SO2; and AC1, 2, 3, 4, 5, and 6 of SO3.

MANAGING THIS ACTIVITY

For this activity calculators will be necessary.

3.1 Completed values are in bold.

Number of satchels		10	16	20	25	30	35	40	45	50
Catering budget per guest (in rands)		**80,00**	**50,00**	**40,00**	**32,00**	**26,67**	**22,86**	**20,00**	**17,78**	**16,00**

3.2 Unlike the similar questions in activities one and two, in this case there is only one method for calculating the budget per satchel in the table: divide the total satchel budget (R800,00) by the number of satchels. It is expected that some students should notice that early on in the table we had 16 in the top line and 50 in the bottom line and later in the table we had 50 in the top line and 16 in the bottom line. (This also happens with 20 and 40). This relationship is true because in the inverse proportion problem (xy = k) the product of the variables is always constant.

3.3 As the number of satchels increases so the budget per satchel decreases. Unlike the functions in the first two activities, here the budget per satchels does not decrease by a constant amount (i.e. the rate is not constant).

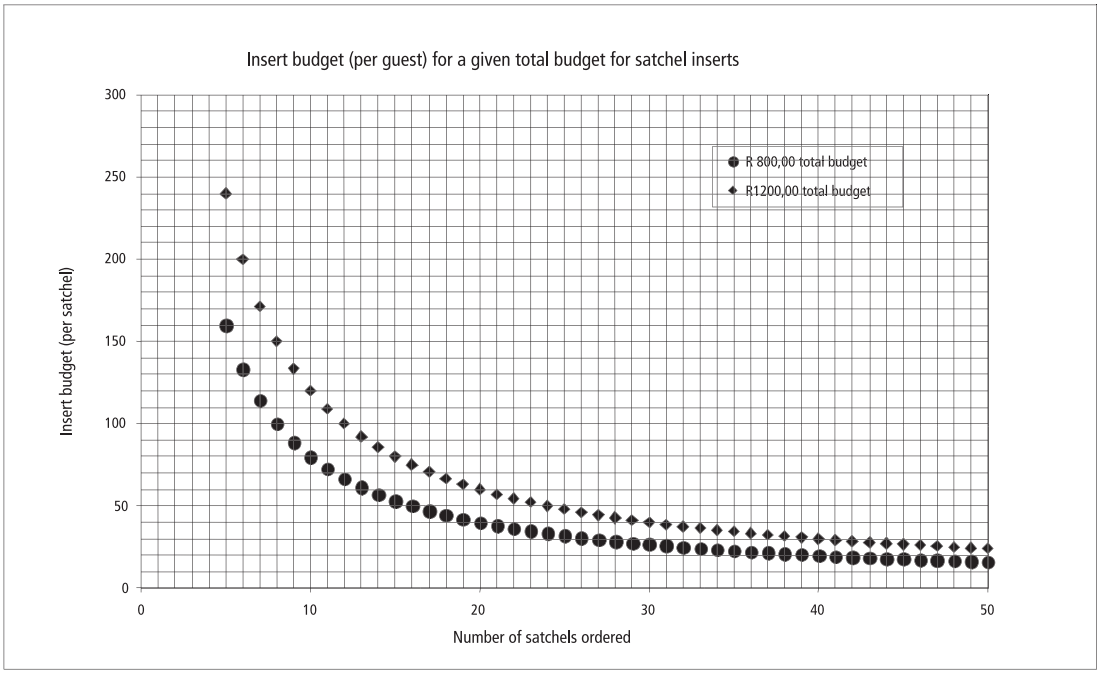
3.4 If 26 people attended the function the budget per satchel would be R30,77. (This is calculated by dividing the total budget by the number of satchels: 800 ÷ 26.)

3.5 If the budget per satchel is R12,50 and there is a total budget of R800,00 then the maximum the number of satchels you could fill is 64. This is calculated as follows:

Since … $\textit{Budget per satchel} = \dfrac{\textit{Total budget}}{\textit{Number of satchels}}$

Then … $\textit{Number of satchels} = \dfrac{\textit{Total budget}}{\textit{Budget per satchel}} \quad \therefore \quad \dfrac{800}{12{,}50} = 64$ satchels

3.6 The answer to question 3.8 is also given on this same set of axes.



3.7 No, it would not make sense to join the points because the data is not continuous. You can only cater for whole numbers of satchels. The key idea for students to grasp here is that with this discrete function the input number must be a whole number.

3.8 See graph in 3.6.

3.9.1 The graph with a total budget of R1 200,00 lies above the original graph with the total budget of R800,00.

3.9.2 SAME: both graphs are decreasing (going down) as the number of satchels increases. For both graphs the budget per satchel is not decreasing at a constant rate. Neither of these graphs can have a value that corresponds to an x-value of zero and similarly neither of them can have an x-value that corresponds to a zero y-value.

DIFFERENT: The graph with the larger constant (R1 200,00) lies above the graph with the smaller constant (R800,00).

3.9.3 No, the budget per satchel does not decrease by the same amount as the number of satchels increases. For example, for a conference with 10 satchels the budget per satchel is R80,00 and for a conference with 20 satchels the budget per satchel is R40,00. Here there is an increase of 10 satchels and a decrease of R40,00 in budget per satchel. However, for a conference with 30 satchels (again an increase of 10 guests) the budget per satchel is now R26,67 – a decrease of only R13,33 and not R40,00. This is because the product of the number of satchels and the budget per satchel is always equal to a constant (the total budget).

3.10 As discussed in 3.5 the formula can be written as:

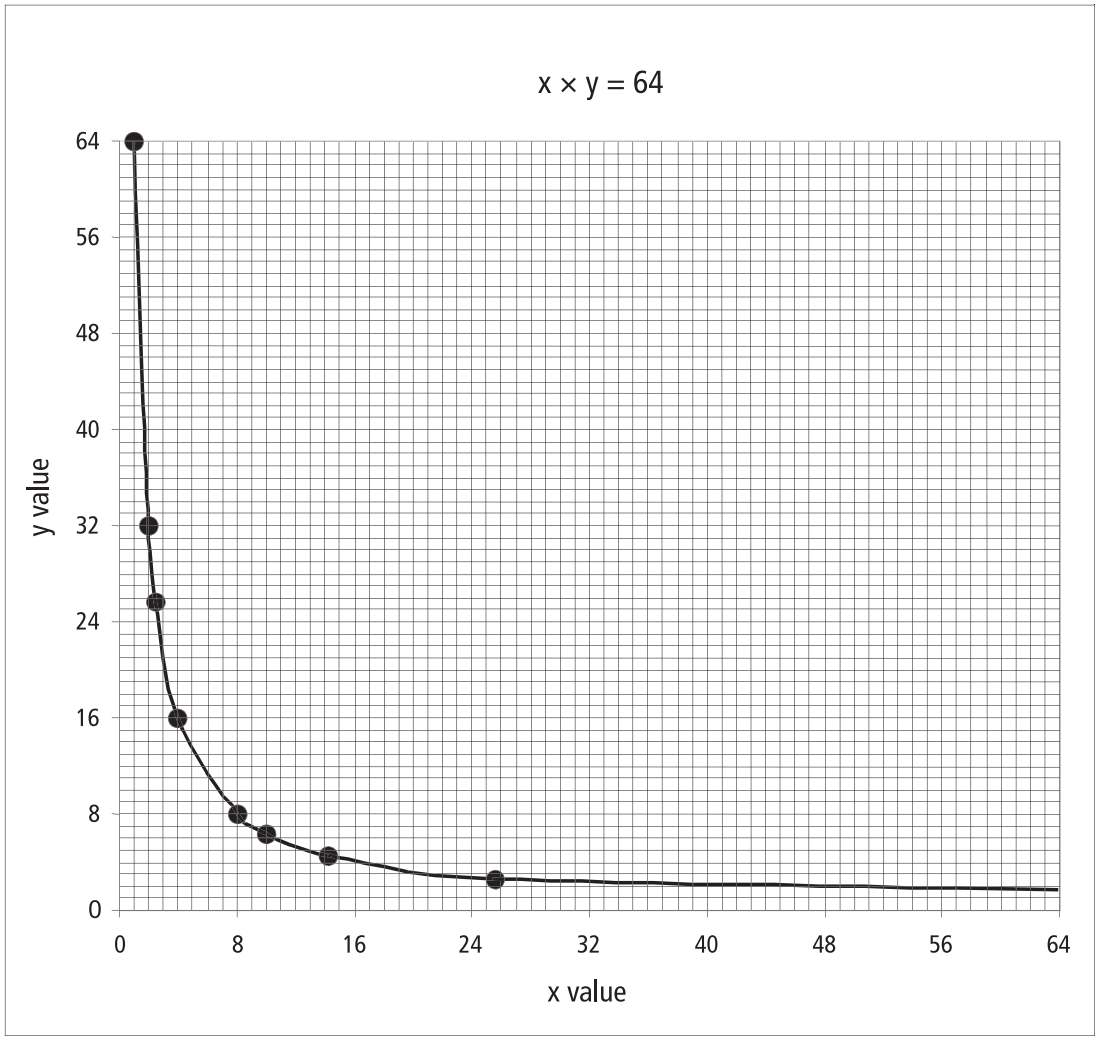
$$\textit{Budget per satchel} = \frac{\textit{Total budget}}{\textit{Number of satchels}}$$

3.11 Completed values in bold

Value of *x*	1	2	2,5	**4**	8	10	**14,2**	**25,6**	128
Value of *y*	**64**	**32**	**25,6**	16	8	**6,4**	4,5	2,5	0,5

3.12 Since the formula given is $xy = 64$, we can substitute the known value (the value that appears in the table) into the formula and then solve (by dividing) for the unknown value. In other words, to calculate the *y*-value we can divide the 64 by the *x*-value and to calculate the *x*-value we can divide the 64 by the *y*-value. There isn't really another way to do this calculation.

3.13



3.14 In the places marked with * the students should fill in the value which they read off the graph.

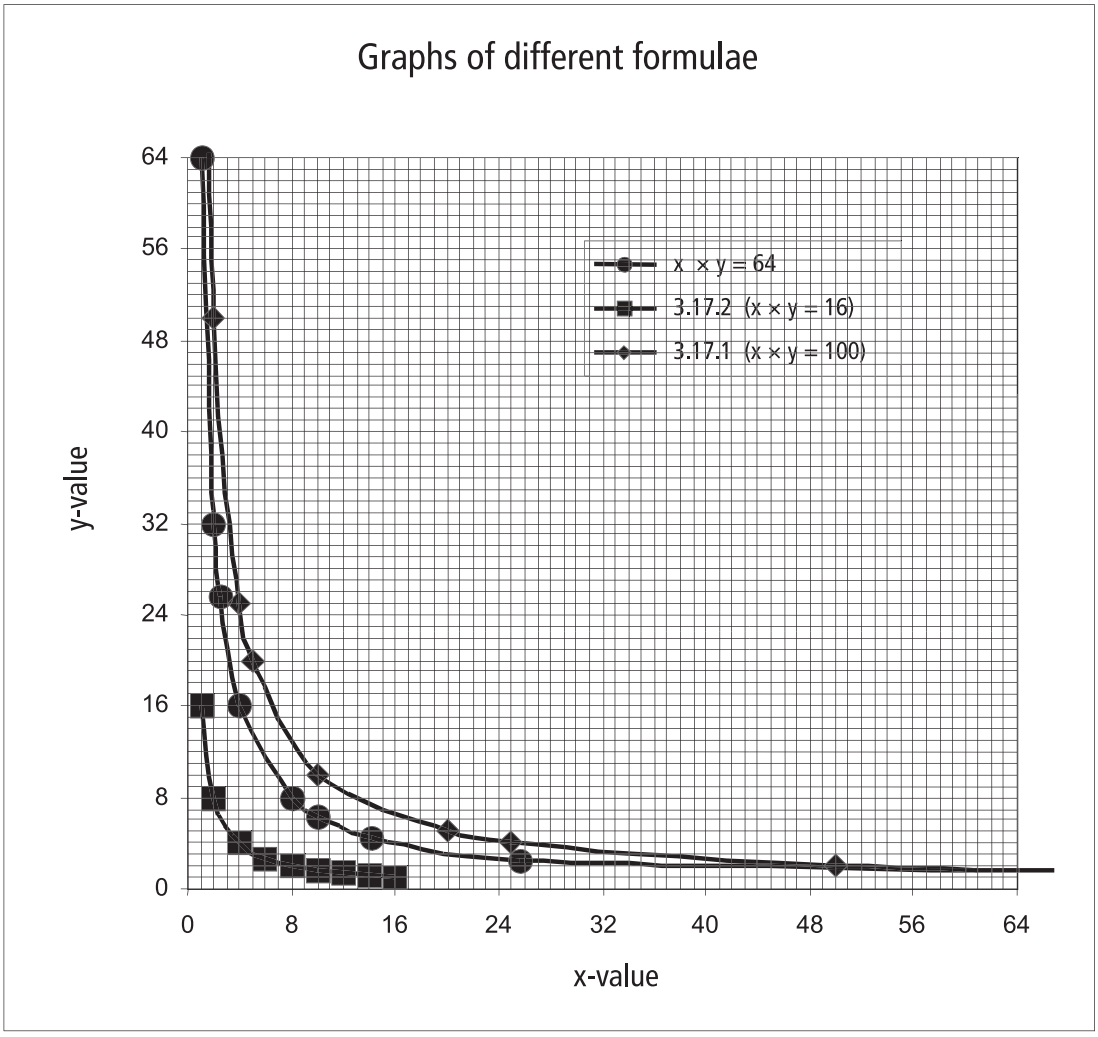
Value of x	3	5,3	7,2 *	8,4 *	16	32 *	46	42,7 *
Value of y	21,3 *	12,1 *	8,9	7,6	4 *	2	1,4 *	1,5

3.15 Although reading a value off a graph is a lot quicker than using a formula, the answer which you can read off a graph is obviously not as accurate as that which you calculate with the formula.

3.16 When x = 0 then the y-value calculated using the formula would be: $\frac{64}{0}$ which is undefined.

Therefore with these graphs there is no value for y when x = 0. Similarly, there is also no x-value when y = 0.

3.17



3.18 SAME: All the graphs are decreasing (going down) as the x-value increases. For all the graphs the y-value is not decreasing at a constant rate. None of these graphs can have a y-value that corresponds to an x-value of zero and similarly none of them can have an x-value that corresponds to a zero y-value.
DIFFERENT: The graphs each have a different constant. The smaller the constant the closer the graph comes to the axes and the origin.

Activity 3 — Inverse proportion

In this activity we will examine a pattern (function) that is quite different from the patterns we have studied so far. If you think back to the first two activities you may have realised that in very general terms we worked with patterns (functions) in which we had two related things that could change (variables), and generally speaking as the one increased so the other increased as well. As the field workers drove further so their travel allowance increased. As people bought more conference satchels so the total cost went up. In this activity we will work with situations in which the relationship between the two variables is different—look out for this difference while at the same time continuing to look for similarities between the pattern studied in this activity and the patterns already studied.

Satchel inserts

One of the services that your conference satchel company offers is to put inserts into each satchel. Satchel inserts can vary from a pencil and some sheets of paper costing only a few rand per satchel to deodorants, sweatshirts, and other very expensive inserts that cost quite a lot per satchel. Typically, the client tells you how much their total "insert" budget is and how many satchels they will need. You then choose items to put into the satchel to match the budget. One of your clients has indicated that they have a total budget of R800,00 for satchel inserts for their conference. They will, however, only be able to tell you in a few days time how many satchels they will need.

From previous experience you know that there will be more than 10 satchels and maybe even as many as 50. To get ahead with planning, you have decided to develop a table that will tell you how much you can spend on each satchel's inserts when the client decides how many satchels they will be ordering.

Number of satchels		10	16	20	25	30	35	40	45	50
Insert budget per satchel (in rands)										

3.1 Copy and complete the table.

3.2 Describe, in words, how you calculated the insert budget (per satchel) in order to complete the table. Can you think of another way that you could have done this?

3.3 Describe, in words, what happens to the insert budget (per satchel) as the number of satchels increases.

3.4 What would the insert budget (per satchel) be if the client ordered exactly 26 satchels?

3.5 From experience you know that it is not really cost-effective for you to place inserts in satchels if the insert budget (per satchel) is less than R12,50. How many satchels could you fill if the insert budget (per satchel) is exactly R12,50?

3.6 Rather than using a table, you have come to realise that graphs can be more useful in determining values such as the insert budget (per satchel). Draw a graph for the situation.

3.7 Look at the graph that you have drawn. Did you join the points on your graph? Give a reason for your answer.

3.8 The client has suddenly told you that the total budget for inserts has been increased to R1 200,00. Draw the graph for the new insert budget (per satchel) onto the same set of axes that you used in 3.5.

3.9 Compare the two graphs:
3.9.1 How can you see from the graphs that the total budget has increased?
3.9.2 How are the two graphs the same and how are they different?
3.9.3 Has the insert budget (per satchel) increased by the same amount per satchel or not? Discuss your answer.

3.10 Write down a formula that can be used to determine the insert budget (per satchel) for a given total budget and number of satchels.

Formulae, graphs, tables and words

In the "satchel inserts" part of this activity we worked with a situation (described in words), developed a table of values, drew a graph and developed a formula to describe the situation. In this part of the activity we will start with the formula, develop a table of values, draw the graph and use the formula, the table and/or the graph to answer questions.

Consider the formula: $x \times y = 64$.
Mathematicians refer to x and y as variables.

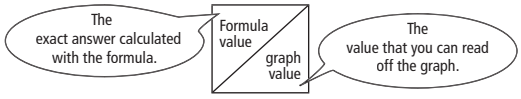
3.11 Copy and complete the table below.

Value of x	1	2	2,5			10			128
Value of y				16	8		4,5	2,5	

3.12 Describe, in words, how you determined missing information in the table. Can you think of another way that you could have done this?

3.13 Draw a graph for the formula by plotting the points in the table and joining them with a smooth curve.

3.14 Copy and complete the table below using both your formula and the graphs that you have drawn. In each case fill the value you calculated using the formula into the top half of the cell and the value you read off the graph into the bottom half of the cell.



Value of x	3	5,3			16		46	
Value of y			8,9	7,6		2		1,5

3.15 Which method of determining the missing value did you prefer? Did you prefer using the formula or using the graph?

3.16 What is the y-value when x = 0?

3.17 On the same set of axes that you used in 3.13 draw two more graphs as follows:
3.17.1 Draw the graph that corresponds to the formula: x × y = 100.
3.17.2 Draw the graph that corresponds to the formula: x × y = 16.

3.18 In what ways are the three different graphs the same? In what ways are they different? Can you relate the differences to differences in the formulae?

Activity 4 — Some other patterns

ABOUT THIS ACTIVITY

In this activity the underlying concept that is addressed is the notion that although there are numbers, there is not by implication a mathematical relationship (rule/function) that relates the numbers to each other. In the first and third of the situations studied in this activity there is some discernable pattern. Our social knowledge of the situations can give us the confidence to make predictions (to interpolate and extrapolate). However, there are no rules/functions that describe the situations. In the second of the situations there isn't even the possibility of using social knowledge of the context in order to make predictions, since the data are completely unrelated.
This activity is aligned with unit standard 9007 and addresses AC 1 and 2 of SO1; AC 1 and 2 of SO2; and AC 3, 5, and 6 of SO3.

MANAGING THIS ACTIVITY

One of the key purposes is to get the ideas across. Some effort should be made to avoid getting into discussions that distract the class from the point at hand.

4.1 Answers are given in the table below (some of the rows of the original table have been deleted to save space):

NYANGA	05:30	05:40	05:50	06:00	06:10	06:15	06:25	06:30	06:35	06:35	06:40	06:40	06:55	07:00	07:10	07:10	07:25	07:30	07:35	07:35	07:40	07:40
MOWBRAY	06:05	06:15	-----	-----	06:45	06:55	-----	-----	-----	-----	07:30	07:15	via	07:45	07:45	07:45	-----	08:05	08:15	-----	08:15	-----
CAPE TOWN	06:30	06:30	06:40	07:00	07:00	07:10	07:10	07:15	07:40	07:40	-----	07:50	08:15	08:30	08:30	08:30	09:00	08:30	08:40	08:40	08:40	08:30
Nyanga to Mowbray	00:35	00:35			00:35	00:40					00:50	00:35		00:45	00:35	00:35		00:35	00:40		00:35	
Nyanga to Cape Town	01:00	00:50	00:50	01:00	00:50	00:55	00:45	00:45	01:05	01:05		01:10	01:20	01:30	01:20	01:20	01:35	01:00	01:05	01:05	01:00	00:50
NYANGA	07:45	08:00	08:20	08:20	08:30	09:00	10:00	10:50	11:10	12:00	13:00	13:00	14:00	14:50	15:30	16:00	16:00	16:20	16:25	17:10	17:15	18:00
MOWBRAY	08:20	08:35	08:55	08:55	09:10	09:35	10:35	11:20	11:45	12:30	13:30	13:40	14:30	15:30	16:20	16:40	16:35	-----	17:05	17:40	-----	18:40
CAPE TOWN	09:00	09:00	09:20	09:20	-----	10:00	10:55	-----	12:00	-----	-----	-----	-----	-----	-----	-----	16:55	17:10	-----	-----	18:00	-----
Nyanga to Mowbray	00:35	00:35	00:35	00:35	00:40	00:35	00:35	00:30	00:35	00:30	00:30	00:40	00:30	00:40	00:50	00:40	00:35		00:40	00:30		00:40
Nyanga to Cape Town	01:15	01:00	01:00	01:00		01:00	00:55		00:50								00:55	00:50			00:45	

4.2.1 The fastest trip from Nyanga to Mowbray takes 30 minutes and the fastest trip from Nyanga to Cape Town takes 45 minutes.

4.2.2 The slowest trip from Nyanga to Mowbray takes 50 minutes and the slowest trip from Nyanga to Cape Town takes 1 hour and 35 minutes.

4.3.1 At this time, the table shows that there are two buses travelling to Mowbray. They travel different routes (indicated by the word "via" on the table).

4.3.2 There is no pattern in the sense of a pattern with a rule/formula, but it is clear from the graph that at certain times of the day (particularly the early part of the morning – from 06:00 to 08:00) the journeys take much longer than at other times of the day. Just because this graph/table involves numbers does not mean that there is a relationship in a mathematical sense, and even though in this case broad patterns can be seen we can't use those patterns to make predictions as we have been able to do in previous instances in the unit.

4.3.3 Yes and no. We can make a guess by looking at an upper and lower limit. The trip must take longer than 40 minutes (the time taken to get to Mowbray) and at 07h00 it took an hour and 35 minutes. The graph seems to suggest that the trip would take as long at this time of the day as it did at 07h00. Therefore our guess would need to be somewhere between 40 minutes and 1hour and 35 minutes. Our concern is, however, not so much with the guesses that the students make and/or how they arrive at this guess, but rather with making the point that it is impossible to predict this answer exactly using the information provided. Although there is a pattern in a very broad sense, there is no rule that defines this pattern.

4.4 No. As stated in the previous question, there is a pattern here, in a very broad sense, but it is not possible to develop a rule or formula that defines this pattern.

4.5 No matter how strongly a student attempts to argue his/her case, in this context it is absolutely impossible to develop a pattern/rule and therefore no answers can be determined.

4.6 This is random data. The age, gender and change for each of the subjects in the tables are completely unlinked and therefore it is impossible to develop a rule or formula that relates these facts to one another.

4.7 Yes. As with the bus, we can make some kind of prediction here. Since Namhla's weight at aged 4 months was 6½ kg and at aged 10 months it was 9¾ kg, it is unlikely that her weight between these ages was much less than 6½ kg and/or much greater than 9¾ kg.

4.8 Even though there is an increasing pattern in the graph, there is no mathematical way of predicting the exact weight. Incidentally, the coffee stain has covered the point in this region, giving us some limits for our estimates (they must be under the stain.)

4.9 SAME: All three of the situations are such that they do not fit a rule/formula.

DIFFERENT: The bus situation and Namhla's road to health chart both show instances where it is possible to make some rough estimates based on the visible patterns, but the age and change situation is one in which the data is completely random and no link or even rough pattern can be made in order to make other predictions.

Activity 4 — Some other patterns

As the title of this activity suggests we are going to explore some other unusual patterns in this activity. You will get a chance to practise many of the skills that you have acquired in the previous activities but you will also have to be attentive to the possibility that not all of what you have already learnt can be applied here.

Bus timetable

The timetable below has been taken from the Golden Arrow website. It shows the departure and arrival times for the buses that travel from Nyanga to Mowbray and from Nyanga to Cape Town from Monday to Friday. There are many things that we can establish from this timetable. To begin with, study the timetable and be sure that you feel confident about how to read it. NOTE: The unshaded rows are not on the original timetable. They have been added for the purpose of this activity.

Golden Arrow Bus Timetable
NYANGA to MOWBRAY to CAPE TOWN
Timetable Number: 004301
Mondays to Fridays
(source http://www.gabs.co.za)

NYANGA	05:30	05:40	05:50	06:00	06:10	06:15	06:25	06:30	06:35	06:35	06:40	06:40	06:55	07:00	07:10	07:10	07:25	07:30	07:35	07:35	07:40	07:40
NY 108	via	via	via	via	via	-----	via	via	-----	-----	via	via	-----	via	-----	via	via	via	-----	via	via	via
NY 3A	-----	-----	-----	-----	-----	-----	-----	-----	-----	via	-----	-----	via	-----	-----	-----	-----	-----	via	-----	-----	-----
GUGULETU	-----	05:50	-----	-----	-----	-----	-----	-----	-----	-----	via	-----	-----	-----	-----	-----	-----	07:40	-----	-----	-----	-----
NY 1	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	via	-----	-----	-----	-----	-----	-----	-----	-----	-----
FREEWAY	-----	-----	via	via	-----	-----	via	via	via	via	-----	-----	-----	-----	-----	-----	via	-----	-----	via	-----	via
MOWBRAY	06:05	06:15	-----	-----	06:45	06:55	-----	-----	-----	-----	07:30	07:15	via	07:45	07:45	07:45	-----	08:05	08:15	-----	08:15	-----
CAPE TOWN	06:30	06:30	06:40	07:00	07:00	07:10	07:10	07:15	07:40	07:40	-----	07:50	08:15	08:30	08:30	08:30	09:00	08:30	08:40	08:40	08:40	08:30
Nyanga to Mowbray																						
Nyanga to Cape Town																						
NYANGA	07:45	08:00	08:20	08:20	08:30	09:00	10:00	10:50	11:10	12:00	13:00	13:00	14:00	14:50	15:30	16:00	16:00	16:20	16:25	17:10	17:15	18:00
NY 108	via	via	via	via	via	Via	via	via	via	via	via	-----	-----	-----	-----	-----	via	-----	Via	via	-----	-----
NY 3	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	via	via	via	Via	via	-----	-----	-----	-----	-----	via
GUGULETU	-----	-----	08:30	-----	-----	-----	-----	-----	-----	via	via	via	via	via	Via	via	16:10	-----	-----	-----	-----	via
FREEWAY	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	via	-----	-----	via	-----
MOWBRAY	08:20	08:35	08:55	08:55	09:10	09:35	10:35	11:20	11:45	12:30	13:30	13:40	14:30	15:30	16:20	16:40	16:35	-----	17:05	17:40	-----	18:40
CAPE TOWN	09:00	09:00	09:20	09:20	-----	10:00	10:55	-----	12:00	-----	-----	-----	-----	-----	-----	-----	16:55	17:10	-----	-----	18:00	-----
Nyanga to Mowbray																						
Nyanga to Cape Town																						

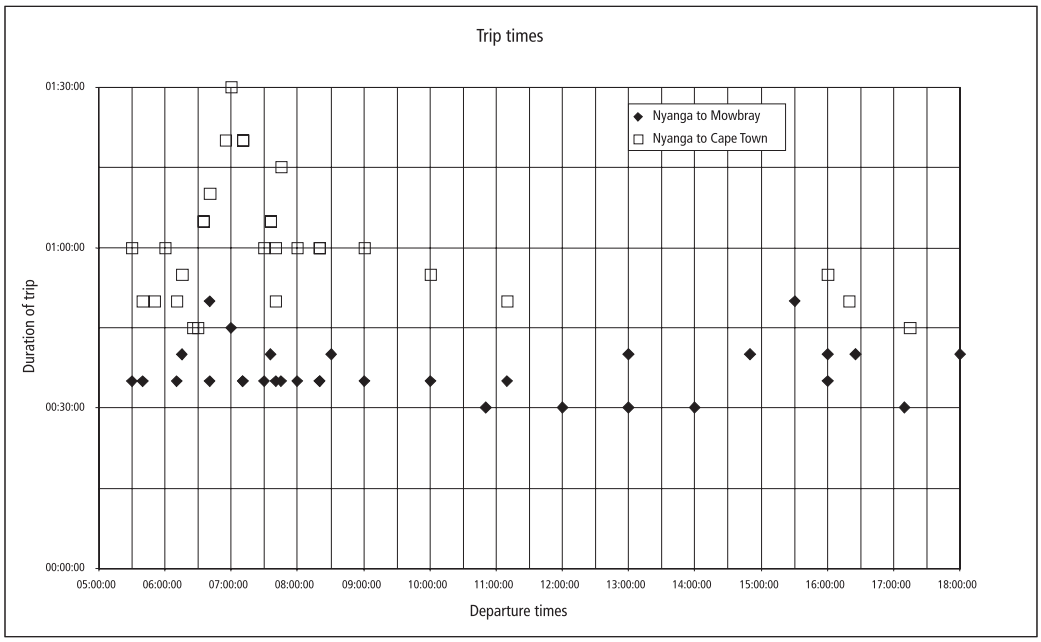
4.1 Calculate the time that it takes each bus to travel from Nyanga to Mowbray and from Nyanga to Cape Town. Fill these times into the spaces provided.

4.2 Determine the following from the table you have filled in:

4.2.1 How long do the fastest trips from Nyanga to Mowbray and from Nyanga to Cape Town take?

4.2.2 How long do the slowest trips from Nyanga to Mowbray and from Nyanga to Cape Town take?

In the activities so far, we have often plotted the points on a graph as another way of looking at information. A graph of the durations you have just calculated is drawn below.



4.3 By looking at either the graph or your table, answer the questions that follow:

4.3.1 How can it be that the graph shows two different durations for the Nyanga to Mowbray trip departing at 16:00?

4.3.2 Is there any pattern that you can see that relates the time of day at which the trip is taken to the duration of the trip?

4.3.3 Can you use the information to predict how long a trip from Nyanga to Cape Town would take at 13:00? And at 15:00?

4.4 Do you think that it is possible to develop a formula that relates the duration of the trip to the time of day at which it is taken? If yes, what is it? If no, why not?

Age and change

The first 10 men and first 10 women who boarded the 06:40 bus from Nyanga to Cape Town were asked how old they were and how much money they had in their pockets. The results of the survey are shown in the table below.

Men			Women		
Name	Age	Change	Name	Age	Change
Andre	57	R 33.00	Delisile	37	R 10.00
Emile	31	R 33.00	Itumeleng	52	
Wayne	33	R 25.00	Gloria	38	R 35.00
Aaron	27	R 3.00	Rose	24	R 7.00
Tony	52	R 33.00	Portia	39	R 32.00
Mbulelo	20	R 31.00	Martha	65	R 6.00
Thabang	35	R 8.00	Thando	51	R 33.00
David		R 11.00	Phindile	25	R 12.00
Jacob	57		Veronica	35	R 13.00
Neil	48	R 11.00	Jo-Anne		R 7.00

4.5 You will notice that the person who completed the research did not complete all of the cells in the table. Taking care to explain your thinking in each case, determine:

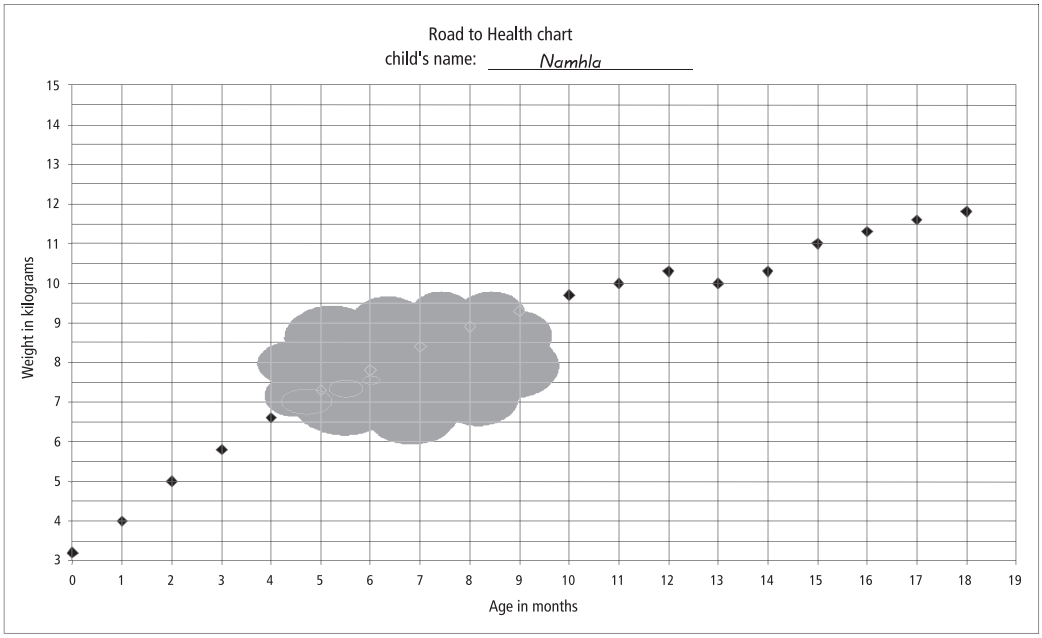
4.5.1 How old David is.

4.5.2 How much change Jacob had in his pocket.

4.5.3 How old Jo-Anne is.

4.5.4 How much change Itumeleng had in her pocket.

4.6 Do you think that it is possible to develop a formula that relates the age and/or gender of the passengers to the amount of change they have in their pocket? If yes, what is it? If no, why not?



Namhla's Road to Health chart

Phindile, who you will remember from the bus journey (yes, the lady with R12,00 in her pocket), was on her way to the clinic with her daughter Namhla. Phindile dropped Namhla's Road to Health Chart on the floor of the bus. The chart is shown above. You will notice that somebody has spilt some coffee on the chart.

4.7 Based on the information available, can you decide on Namhla's weight at 5, 6, 7 and 8 months? Explain how you determined your answers.

4.8 Do you think that it is possible to develop a formula that relates Namhla's weight to her age in months? If yes, what is it? If no, why not?

4.9 Look back at the three situations we have examined in this activity. In what ways are they the same? In what ways are they different?

Activity 5 — Yet another pattern

ABOUT THIS ACTIVITY

In this activity we look at functions which behave differently from those studied in the previous activities in that they are not smooth curves. Students are exposed to interesting situations where we look at functions which are a combination of a constant and a step function (i.e. within each of the clearly visible sections/intervals of the function, it is constant).

This activity is aligned with unit standard 9007 and addresses AC 1, 2 and 3 of SO1; AC 1, 2 and 3 of SO2; and AC1, 2, 3, 4, 5, and 6 of SO3.

MANAGING THIS ACTIVITY

This is a straight forward activity based on common everyday situations and students should find it uncomplicated.

5.1.1 08h00 till 10h30 is a period of 2 – 3 hours and therefore you would have to pay R6,00.

5.1.2 08h45 till 11h52 is a period of 3 – 4 hours and therefore you would have to pay R8,00.

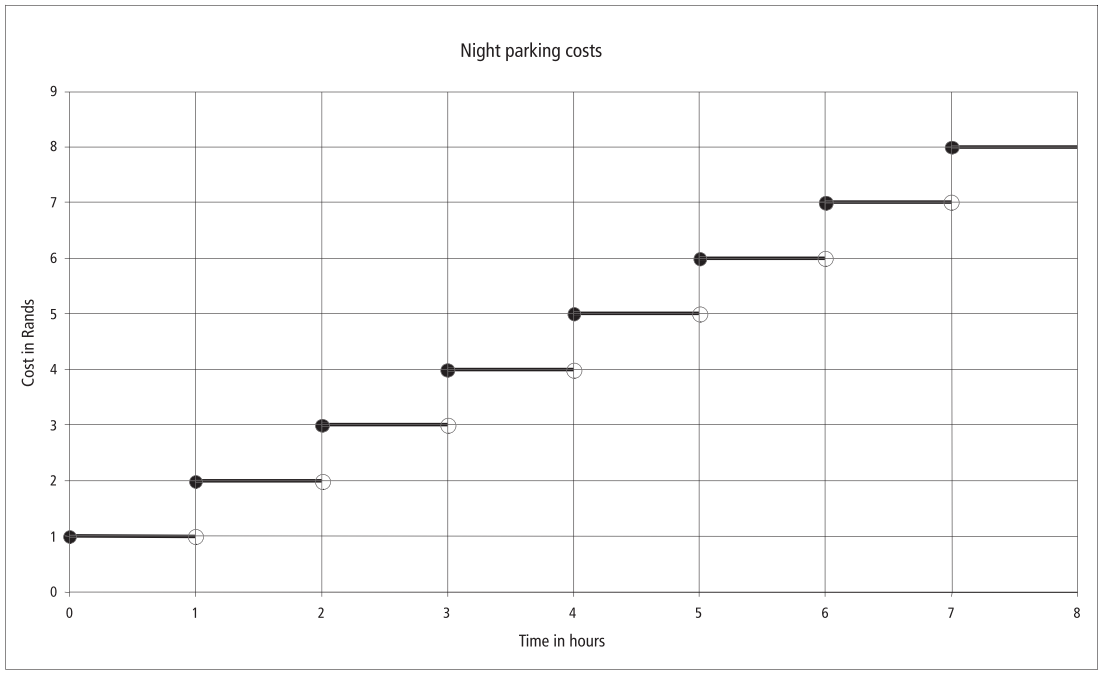
5.1.3 09h35 till 16h18 is a period of 6 – 7 hours and therefore you would have to pay R30,00.

5.1.4 19h00 till 23h45 is a period of 4 – 5 hours but since you have entered after 18h00 you are charged the night parking rate of R1,00 per hour. Therefore you would have to pay R5,00.

5.1.5 16h24 till 22h19 is a period of greater than 4 – 5 hours and therefore you would have to pay R15,00.

5.2 The points which are coloured in are elements of the solution set. The points which are not coloured in are not elements of the solution set. However, the open points (ones not coloured in) are necessary to mark the end of the line that precedes each one. Notice how for some x-values there are two points – one that is coloured in and one that is not. It is at these x-values that the y-value changes (steps up – these functions are known as step functions).

5.3



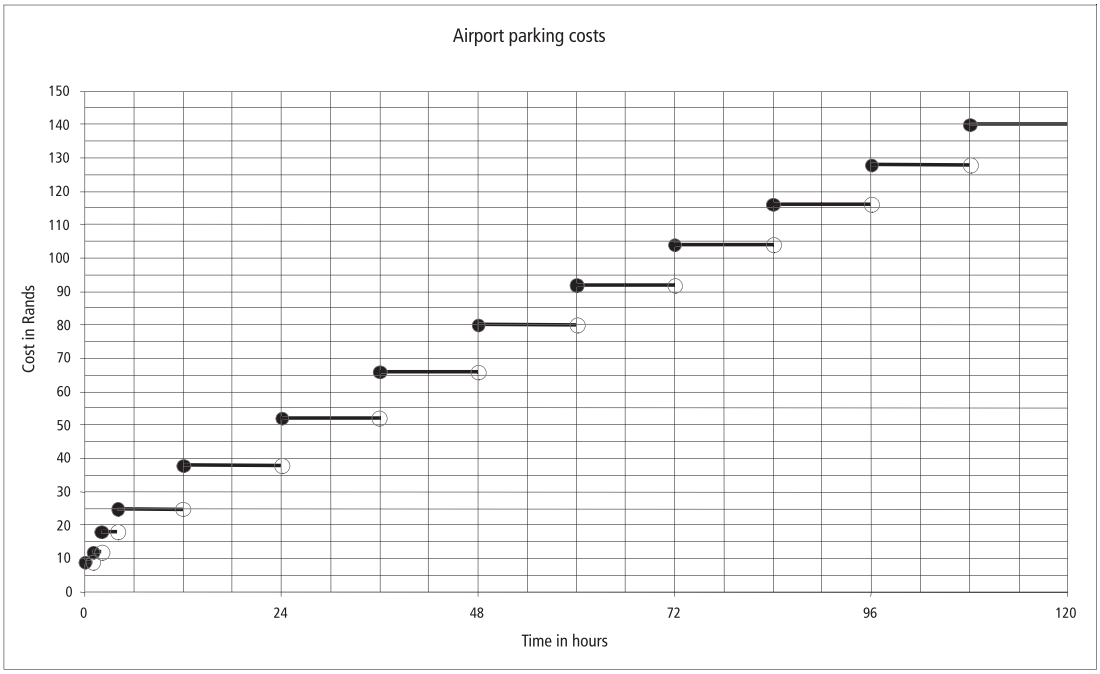
5.4 In this context it is merely a case of preference and there is no clear advantage to either option.

5.5 Yes there is, but there is no useful formula in this case.

Mathematically, it would look like this:

$$Cost\,(x) = \begin{cases} 4 & x < 2 \\ 6 & 2 \le x < 3 \\ 8 & 3 \le x < 4 \\ 15 & 4 \le x < 5 \\ 20 & 5 \le x < 6 \\ 30 & 6 \le x < 7 \\ 35 & 7 \le x < 8 \\ 50 & x > 8 \end{cases}$$

5.6



5.7 SAME: They are all made up of little "steps".

DIFFERENT: In the second shopping centre graph the "steps" are constant and in the other two this is not the case.

5.8 As with question 5.5 there isn't a useful formula to use here.

5.9 Shoe size, annual salary etc.

Activity 5 — Yet another pattern

Before we try to consolidate our exploration of patterns, we will consider one last pattern (for now). As you work through this activity, try hard to see the relationship between this pattern and all of the other patterns we have explored so far.

Parking rates
The parking tariffs table alongside appears at the entrance to the parking lot of a popular shopping centre in Cape Town.

PARKING TARIFFS

Hours	Cost
0 – 2	R 4,00
2 – 3	R 6,00
3 – 4	R 8,00
4 – 5	R 15,00
5 – 6	R 20,00
6 – 7	R 30,00
7 – 8	R 35,00
8 +	R 50,00
Night parking (enter after 18h00 and exit before 06h00)	**R 1,00** per hour

5.1 How much would you have to pay for your parking if you stayed in the parking lot for each of the following periods of time:

5.1.1 From 08h00 till 10h30?

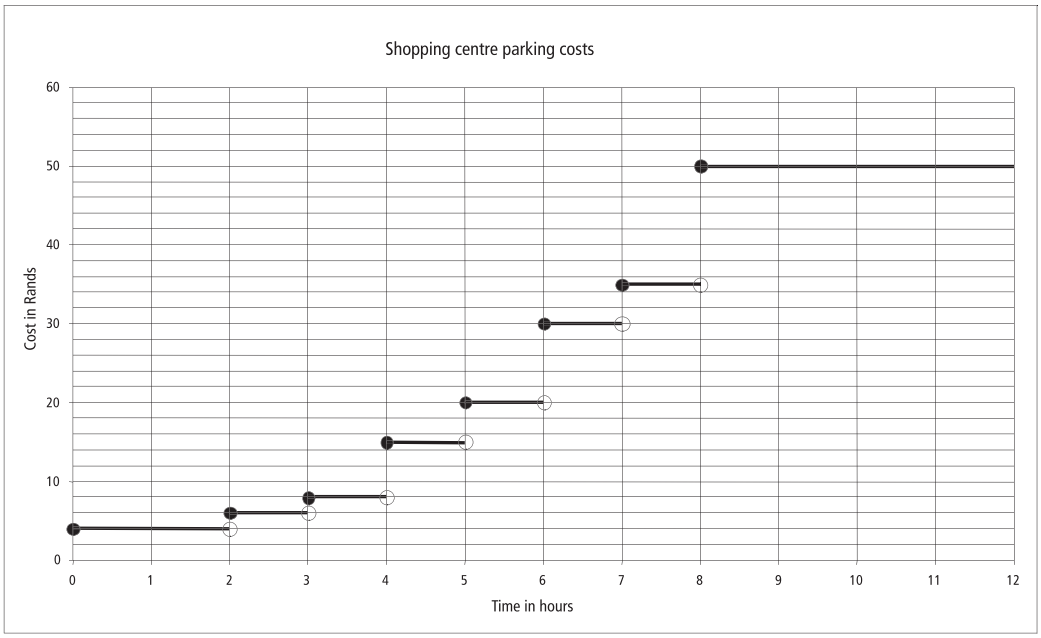
5.1.2 From 08h45 till 11h52?

5.1.3 From 09h35 till 16h18?

5.1.4 From 19h00 till 23h45?

5.1.5 From 16h24 till 22h19?

The graph below depicts the same information.



5.2 Answer the following questions with respect to the graph:

5.2.1 There are two kinds of points on the graph: points that have been coloured in and others that are "open." Why are there these two types of points?

5.2.2 There are lines that join some of the points but not all of the points have been joined by lines. Why is this so?

5.2.3 Does the price for parking increase uniformly or not? How can you see this from the graph?

5.3 The graph that has been drawn for you does not deal with night parking. Develop your own graph for night parking. How is this graph similar to the day-time parking graph? How is it different from the day-time parking graph?

5.4 Which representation, the graph or the table, conveys the cost of parking most effectively?

5.5 Do you think that it is possible to develop a formula that relates the time spent in the parking garage to the cost? If yes, what is it? If no, why not?

Airport parking

The sign alongside is displayed at the entrance to the Cape Town International Airport parking lot.

PARKING TARIFFS

Hours	Cost
0 – 1	R 9,00
1 – 2	R 12,00
2 – 4	R 18,00
4 – 12	R 25,00
12 – 24	R 38,00
24 – 36	R 52,00
36 – 48	R 66,00
48 – 60	R 80,00

Thereafter R12,00 for every additional 12 hours or part thereof

5.6 Develop a graph that could be used to determine the cost of parking for any period between 0 hours and five days (120 hours).

5.7 In what ways is the graph that you have just drawn different from the two shopping centre graphs? In what ways is it the same? Discuss.

5.8 Do you think that it is possible to develop a formula that relates the time spent in the parking garage to the cost? If yes, what is it? If no, why not?

5.9 Can you think of any other situations—particularly situations in your own life—that could be described by a graph that is similar to the ones we have seen in this activity? Describe these situations in detail.

Activity 6 — A pattern with a twist

ABOUT THIS ACTIVITY

In this activity we work with functions of the form $y = ax + b$, where $a < 0$ (both discrete and continuous). Students work with tables and graphs of these functions and are encouraged to compare them with the other functions studied thus far in the unit.

This activity is aligned with unit standard 9007 and addresses AC 1, 2 and 3 of SO1; AC 1, 2 and 3 of SO2; and AC1, 2, 3, 4, 5, and 6 of SO3.

MANAGING THIS ACTIVITY

Encourage the students to work on their own in this activity as, although new functions are dealt with, much of the content is a consolidation of the ideas and methods already experienced in the other activities.

6.1

Weight of full bottle	1,800kg
Weight of empty bottle	1,050kg
Weight of whisky in full bottle	0,750kg

Volume of bottle	750ml
Volume of a tot	25ml
Number of tots in a bottle	30

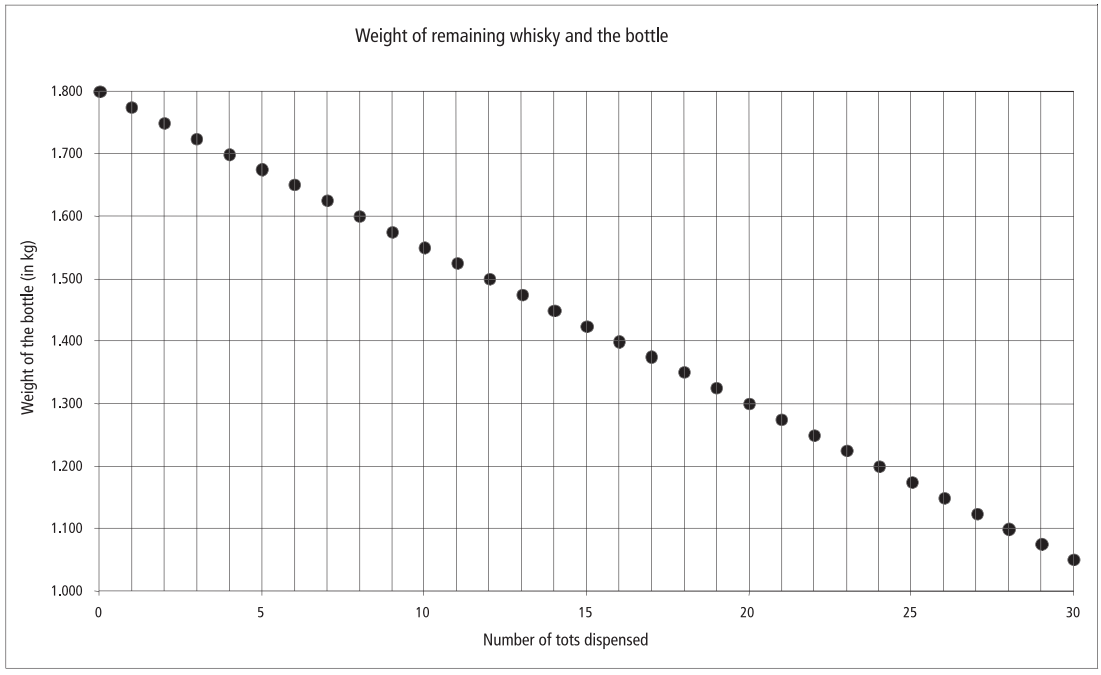
Weight of a single tot	0,025kg

Number of tots dispensed	1	2	3	4	5	10	15	20	25
Expected weight of the bottle (in kg)	1,775	1,750	1,725	1,700	1,675	1,550	1,425	1,300	1,175

6.2 The weight of the bottle would be 1,350kg if 18 tots had been dispensed.

6.3 Since the bottle would weigh 1,500kg after 12 tots have been dispensed and 1,475kg after 13 tots have been dispensed we cannot be sure how many tots have been dispensed. Perhaps in this context there has been some spillage or misuse of the whisky.

6.4



6.5 In this case the points have not been joined as the liquid is dispensed in whole tots only.

6.6.1 (a) If the weight of the empty bottle was greater than 1,050kg, but the volume remained the same, then the new graph would lie above and parallel to the existing graph.
(b) If the weight of the empty bottle was less than 1,050kg, but the volume remained the same, then the new graph would lie below and parallel to the existing graph.

6.6.2 If the bar used an imperial tot measure of 34ml (which is larger than the existing 25ml example) then the graph would have a steeper downward slope – i.e. a greater rate of change.

6.7 Weight of the bottle (in kg) = 0,025 × number of tots dispensed = 1,800

6.8 The weight of the bottle of whisky decreases as the number of tots dispensed increases. We can see this in the table by noticing that the bottom row of numbers gets smaller, and we can see this on the graph by noticing that the graph decreases (goes down) from left to right.

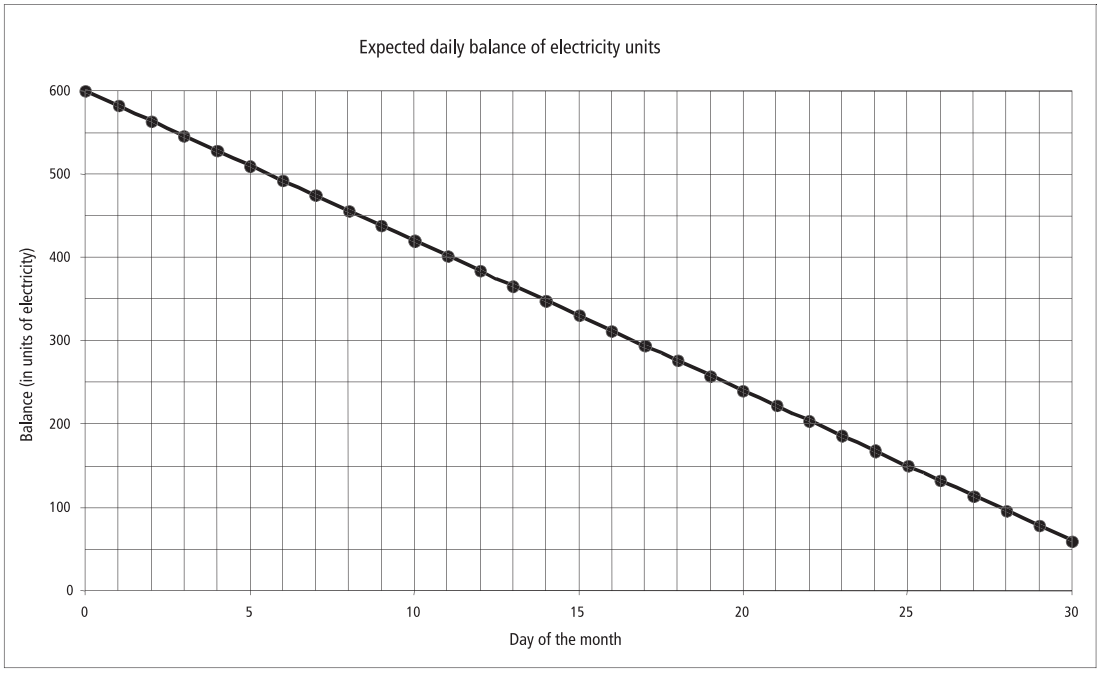
6.9

Number of days	1	2	3	4	5	10	15	20	25
Expected balance of units of electricity	582	564	546	528	510	420	330	240	150

6.10 After 18 days Mary would expect to have 276 units of electricity remianing.

6.11 After 22 days Mary would expect to have 204 units of electricity remaining. Therefore, if she finds that she has only 172 units remaining then she is not on target but is using up more electricity than she had hoped to by this stage. Her rate of usage is higher than she budgeted for.

6.12



6.13 Yes. The points have been joined here because a measurement (of how many units of electricity are left) can be made at any time during the month, not just after whole numbers of days. In other words, the data is continuous.

6.14.1 (a) If Mary started with more than 600 units then the graph would lie above and parallel to the existing graph.
(b) If Mary started with less than 600 units then the graph would lie below and parallel to the existing graph.

6.14.2 If she decides to budget for an increased number of units per day but does not increase the starting amount (i.e. the y-intercept is unchanged), then the rate of decrease would be greater—i.e. the graph would slope down more steeply.

6.15 Expected daily balance (in units) = 600 – 18 × the number of days since loading the dispenser..

6.16 The expected daily balance decreases as the number of days increases. We can see this in the table by noticing that the bottom row of numbers gets smaller, and we can see this on the graph by noticing that the graph decreases (goes down) from left to right. We can also see this from the equation because of the negative sign in front of the rate (which was 18 in this case).

6.17 SAME: Both of the situations in this activity were decreasing functions with a constant rate (functions of the form $y = ax + b$; where $a < 0$).

DIFFERENT: They both had different rates of change and different initial values. The difference in rates of change can be seen in the different slopes and the different initial values can be seen in the different y-intercepts (where the graph cuts the y-axis). Also, the whisky graph has only whole number independent variable values (numbers of tots) therefore it is a discrete function, while with the electricity example the independent variable can be any number, therefore it is a continuous function.

6.18 The purpose of this question is to begin to cue us into activity seven, where we look more closely at the differences and similarities of the functions studied thus far in this unit. Students should at this stage already be able to group some of the functions together, identifying their different and similar properties.

Activity 6 — A pattern with a twist

In this activity we will look at a pattern that we have met in earlier activities but this time it comes with a twist—that is, it behaves slightly differently. Look out for this twist and see what you can apply from the earlier work while at the same time seeing how things are different.

Stock control

One of the key issues in running the bar in a hotel, restaurant or B&B is the management of stock. More money is lost through theft in this area of the business than any other. Some of the reasons for this are that alcohol is both expensive and that its dispensing is less easy to manage than, say, occupancy of rooms. Hotels and restaurants therefore need to establish mechanisms for controlling their stock. The simplest (if not necessarily most effective) is for the manager to reconcile sales and stock at the end of a day or session. Consider a bottle of whisky. The manager can see from the till printout how many tot measures of whisky were sold and knowing the stock that was in the bar at the start of the shift can work out what stock to expect at the end of the shift.

To check the actual stock against the expected stock the manager needs a way of measuring the volume of the bottle without necessarily emptying it (this leads to spillage and wastage). A common technique that is used involves weighing both a full whisky bottle and an empty whisky bottle. By doing this the manager can calculate the weight of the whisky. Knowing how many tots there are in a full bottle of whisky, he can determine what the weight of the bottle and the remaining tots should be at the end of a shift.

A particular bottle of whisky weighs 1,800kg full and 1,050 kg empty. It holds a volume of 750ml. A standard tot measure has a volume of 25ml.

To avoid calculating the expected weight of the bottle after each shift the manager has started to develop the table below.

Weight of full bottle	1,800kg		Volume of bottle	750ml
Weight of empty bottle	1,050kg		Volume of a tot	25ml
Weight of whisky in full bottle			Number of tots in a bottle	

Weight of a single tot	

Number of tots dispensed	1	2	3	4	5	10	15	20	25
Expected weight of the bottle (in kg)									

6.1 Copy and complete the table.

6.2 What would the weight of the bottle be if 18 tots had been dispensed?

6.3 On a given day the manager weighs the bottle and finds that it weighs 1,495kg. How many tot measures had the barman dispensed?

6.4 Once again (as we have done so often in these activities), draw a graph of the situation.

6.5 Look at the graph that you have drawn. Did you join the points on your graph? Give a reason for your answer.

6.6 How would you expect the graph to change if:
6.6.1 The weight of the full bottle of whisky was (a) greater and (b) less than the bottle in the example?
6.6.2 The bar used an imperial tot measure which has a volume of 34ml?

6.7 Write down a formula that can be used to determine the weight of the bottle after a given number of tots have been dispensed.

6.8 Describe, in words, what happens to the weight of the bottle of whisky as tots of whisky are dispensed. How is this reflected in the table, the graph and the formula?

Prepaid electricity

Homes and businesses in South Africa are increasingly making use of prepaid electricity. Units of electricity are bought from an agent (e.g. supermarket) who gives the buyer a slip of paper with a code. This code is entered into the dispenser in the building and electricity is supplied until the units run out. The dispenser has a display that shows the balance of units at any point.

Mary buys units of electricity at the start of the month. She buys enough units so that the balance on her dispenser and the new units together give her 600 units for the month. She does not want to use more than this. In order to "make it to the end of the month" she budgets herself 18 units of electricity per day. Mary has developed a table to help monitor her daily usage.

Number of days	1	2	3	4	5	10	15	20	25
Expected balance of units of electricity									

6.9 Copy and complete the table.

6.10 What balance of units of electricity would Mary expect after 18 days?

6.11 After 22 days Mary notes that the reading on the dispenser is 172 units. Is she on target? That is, is she using electricity at the rate for which she has budgeted?

6.12 This is almost the last graph in this unit! Please draw a graph of the situation.

6.13 Look at the graph that you have drawn. Did you join the points on your graph? Give a reason for your answer.

6.14 How would you expect the graph to change if:
6.14.1 Mary started the month with a balance that was (a) greater than 600 units or (b) less than 600 units (assuming that she still budgets for 18 units per day)?
6.14.2 She decides to budget for an increased number of units per day, say 25 units?

6.15 Write down a formula that Mary can use to determine the expected daily balance for a given budgeted daily consumption.

6.16 Describe, in words, what happens to the expected daily balance as the number of days increases. How is this reflected in the table, the graph and the formula?

6.17 Reflect on the two situations in this activity. How are they the same? How are they different?

6.18 Think back to the other patterns studied in this unit. Which of the earlier patterns are most similar to the pattern in this activity? In what ways are they similar? In what ways are they different?

Activity 7 — Patterns within patterns

ABOUT THIS ACTIVITY

Over the last six activities we have explored a large number of different situations and ways of describing them. We have seen how the same situation can be represented by means of words, tables, graphs and formulae and we have translated from one representation to another.

In this activity we will formally introduce vocabulary to describe what we have explored in the absence of such vocabulary.

Please be aware that the definitions that follow are "fit-for-purpose" definitions. That is, they are adequate for our purpose and help us make sense of the patterns that we have been studying. Mathematicians might want to be more formal in some of the cases—such formality would, however, not help us here.

This activity is aligned with Unit Standard 9007 and addresses AC 1 of SO1; AC 1, 2 and 3 of SO2 and AC 2, 3 and 6 of SO3.

MANAGING THIS ACTIVITY

This activity could be treated as either a self-study or as a cap-stone discussion lesson.
Students will need to be given the full activity and time to make sense of the definitions provided there. It is important that students continuously "look back" over the activities in this unit to make sense of the definitions. It is anticipated that students will be encouraged to read the definition, reflect on it and then search for examples of the concept in the activities of the unit.

The solution set is provided below.

7.1	**Discrete functions**		**Continuous functions**		**Patterns that cannot be described by rules**
Constant functions: $y = a$			Shopping centre parking Airport parking		
Linear functions: $y = ax + b$	**increasing**	**decreasing**	**increasing**	**decreasing**	
	Tables and chairs: $y = 2x + 2$	Stock control: $y = -0{,}025x + 1{,}8$	Conference calls: $y = 5x + 50$	Prepaid electricity: $y = -18x + 600$	
Direct proportion functions: $y = ax$	Conference satchels: $y = 24x$ Lydia's sweatshirts: $y = 30x$		Petrol consumption: $y = 1{,}5x$		
Inverse proportion functions: $xy = k$	Satchel inserts: $xy = 800$		$xy = 64$		
Other					Bus timetable Age and change Namhla's road to health chart

Activity 7 — Patterns within patterns

Over the last six activities we have explored a large number of different situations and ways of describing them. We have seen how the same situation can be represented by means of words, tables, graphs and formulae and we have translated from one representation to another.
In this activity we will formally introduce vocabulary to describe what we have explored in the absence of such vocabulary.

Please be aware that the definitions that follow are "fit-for-purpose" definitions. That is, they are adequate for our purpose and help us make sense of the patterns that we have been studying. Mathematicians might want to be more formal in some of the cases—such formality would, however, not help us here.

Function
In mathematics we use the word *function* for the more everyday word "rule." In the situations we have been working with in this unit there have usually been two sets of numbers that are related to each other by means of a rule. In the "prepaid electricity" situation a rule related the *number of days* to the *expected balance of units of electricity*. The more formal name for the two things related by the rule is *variable*. The number of days is called the *independent variable* while the expected balance of units of electricity is called the *dependent variable*—its value is determined by (dependent on) the number of days.

An important quality of a function that sets it aside from the more everyday term rule is that a rule is called a function if there is only one possible dependent variable (by convention we refer to this as y and write the letter in italics) for each possible independent variable (by convention we refer to this as x and write the letter in italics).

Continuous functions
A *continuous function* is, for our purposes, a function that "makes sense" for all possible values of the *independent variable*. Consider the shadow cast by a tree. At every single instant of the day (while the sun is in the sky), the tree has a shadow and we can develop a rule that relates the length of the shadow (*dependent variable*) to the time of the day (*independent variable*). No matter what time of day you ask about, we can calculate the length of the shadow.

In the *continuous function* situations we have studied we used lines (curves) to "join the points" on the graphs of these functions.

Discrete functions
By contrast to a continuous function, a *discrete function* is, for our purposes, a function that "makes sense" for certain *independent variables* only. Consider the cost of buying packets of sugar. The total cost (*dependent variable*) is determined by the number of packets (*independent variable*) of sugar that we buy. Since sugar can only be bought in whole packets, the only numbers that "make sense" as independent variables are whole numbers and not all of the fractions in between.

In the *discrete function* situations we have studied we decided not to use lines (curves) to "join the points" on the graphs of these functions—recognising that not all of the points on such lines would "make sense."

Patterns that cannot be described by rules or as function
In the situations that we have studied in this unit we came across some that involved numbers within some of which we could even see "patterns" (for example the earlier buses took longer than the later ones). However, the numbers could not be related to each other by means of a rule. We must take care not to fall into the trap of believing that all situations with numbers can be described using a rule!

"Families" of functions
As you studied the situations in this unit it is hoped that you began to see "patterns within the patterns"—different situations that behaved similarly to others. This was by design. The power of mathematics lies in recognising the features that make different situations similar. We can talk about "families" of functions. In this unit we have worked with several "families." Their characteristics are summarised below. It is important to realise that there are many more "families of functions" in mathematics.

Constant function: y = a
Constant functions are functions for which the *dependent variable* is constant or fixed. The "parking lot" situation can be described by a series of *constant functions*. Between 3 and 4 hours the cost of parking is fixed—R8,00. No matter how long you park your car (the *independent variable*); 03:01, 03:25 or 03:58, the cost (the *dependent variable*) remains the same—constant. The general equation (we have used the word formula in this unit) for a *constant function* is given by y = a, where "a" is the *constant*.

Linear function: y = ax + b
Linear functions are functions whose graphs form straight lines.

The general equation for a linear function is given by $y = ax + b$.

"a" is the *rate of change*. You will have noticed that the value of "a" determines the slope (or gradient) of the straight line graph (upward from left to right for $a > 0$ and downward from left to right for $a < 0$).

"b" is the constant. You will have noticed that "b" corresponds to the value of the *dependent variable* when the *independent variable* is 0. In terms of the graph of the linear function, "b" corresponds to the value on the vertical axis (*dependent variable*) where the graph intersects the axis.

Direct proportion function: y = ax
A very special linear function is the linear function with b = 0. The graph of this function goes through the *origin* (the point where the value of both the independent variable and the dependent variable is zero).

$y = ax$ describes situations in which the *independent variable* and the *dependent variable* are in direct proportion—their ratio is constant. In simple terms, y varies as x varies. As x increases so does y and as x decreases so does y.

Inverse proportion function: xy = k
We say that y is *inversely proportional* to x if as x increases so y decreases and as x decreases so y increases. More specifically, the product of x and y is constant. In activity 3 we considered situations that could be described using equations or rules of the form $xy = k$ —inverse proportion situations.

7.1 On the next page you will find a table. By working back through your notes and the activities of this unit, complete the table by writing a title for each activity (e.g. electricity units) as well as the formula/equation/function that you wrote for that situation.

If you can think of other features of the situation, (for example properties of its table, its graph and/or its formula/equation/function) you should write these down—these notes are for you to use to reflect on what we have learnt in this unit!

	Discrete functions		Continuous functions		Patterns that cannot be described by rules
Constant functions: $y = a$					
Linear functions: $y = ax + b$	**increasing**	**decreasing**	**increasing**	**decreasing**	
Direct proportion functions: $y = ax$					
Inverse proportion functions: $xy = k$					
Other					